Investigating a Preservice Secondary School Teacher's Mathematical Knowledge for Teaching Equations

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Abstract

This qualitative study investigated a preservice secondary school teacher's mathematical knowledge for teaching equations (MKT). The study was carried out with Mr. Mwati (Pseudonym). Data was generated using an open-ended paper and pencil test and interviews. I analysed the data using thematic analysis. Analysis of the results indicates that Mr. Mwati demonstrated some evidence of knowledge of solving equations. The findings also reveal that Mr. Mwati's knowledge of students was intertwined with his knowledge of subject matter and knowledge of pedagogy. Limited specialised content knowledge, limited knowledge of content and students as well as limited knowledge of content and teaching were also evident. This study is believed to inform preservice teacher educators about the content of mathematics teacher preparation.

Key Words: *Mathematical Knowledge for Teaching, Preservice Teacher, Secondary School, Equations*

Introduction

The vision of the education sector in Malawi is to be a catalyst for socio-economic development and industrial growth. Malawi's mission is to provide quality and relevant education to the Malawi nation. The purpose of secondary education in Malawi is to prepare students for further education in keeping with their abilities and aptitudes. In implementing the vision and mission, Malawi is focusing on quality in both primary and secondary education. The quality of education in Malawi has been compromised by access since the introduction of Free Primary Education in 1994 (Ministry of Education, 2008).

In Malawi, examination results are regarded as one of the standard measurements of quality of education. Malawi Ministry of Education (2008) reports that, the overall performance of students in secondary schools is poor. The National Education Sector Plan for 2007-2018 (Malawi Ministry of Education: 2008) adds that secondary school students experience poor learning achievement with only around 50 percent of students passing end-of-cycle examinations. In following up this problem in mathematics, I analysed Malawi National Examinations Board Chief Examiners' Reports for years 2008 to 2013. The major finding for this analysis was students' poor performance in Algebra and Geometry. Considering that algebra is the foundation of higher mathematics, I followed up students' performance in algebra. The results indicate overwhelming failure due to misconceptions and errors. For instance, students were unable to formulate and solve equations. When solving quadratic equations, students were unable to change the quadratic equations to standard form. For those who managed to change the equations to standard form, they could not solve for the unknown. When given simultaneous equations, students could not make one unknown subject in the equation when they attempted to use substitution. Those who attempted to use elimination method, say in an equation like $x+3y=11$, could completely eliminate $3y$ and remained with $x=11$. Despite the recommendations the chief examiners put up every year, the problem of poor performance in is still existing (Malawi National Examinations Board Chief Examiners' Reports, 2008-2013).

The problems outlined above were a motivation for the current study. The problems indicate a need for a focus on mathematics teacher education and the algebraic concept of equations. Even (1993) suggests that an important step in improving teaching and learning should be better subject matter preparation for teachers. In order to improve students' performance in mathematics in general, the teacher should enhance profound understanding and acquisition of algebraic concepts and thinking skills. In addition, it is important that preservice mathematics teachers understand the concept of equations, to be better equipped for effective teaching of secondary school mathematics. Thus, this study is believed to inform preservice teacher educators about the content of mathematics teacher preparation. The overall purpose for this study was to investigate a preservice secondary school teacher's mathematical knowledge for teaching equations. In this paper, I show how task-based interviews were used to identify Mr. Mwati's mathematical knowledge for teaching equations. Hence this investigation examined the following question: What mathematical knowledge for teaching equations is displayed by a Malawian preservice secondary school mathematics teacher?

Theoretical Framework

Two constructs guide the theoretical framework for this study; mathematical knowledge for teaching and equations. The theoretical perspective for this study is based on the mathematical knowledge for teaching model which was developed by Ball, Thames, & Phelps (2008). This framework is used to categorise and assess the knowledge needed for teaching mathematics. The mathematical knowledge for teaching framework is a practice based theory (Ball et al., 2008). It focuses on the kind of professional knowledge of mathematics which is different from that demanded by other mathematically intensive occupations. It moves beyond the limiting boundaries of knowledge to include skills, reasoning, habits of mind and sensitivities needed to teach mathematics effectively (Ball et al., 2008).

The mathematical knowledge for teaching (MKT) model elaborates the use of the Shulman's (1986) subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curriculum knowledge categories, organising and defining them in a different way. According to Shulman (1986), the SMK of teachers is "the amount and organisation of knowledge (of a subject) per se in the mind of the teacher". PCK is the most powerful analogies, illustrations, examples, explanations and demonstrations – in a word, the ways of representing and formulating the subject that makes it comprehensible to others (Shulman, 1987, p.8). There are six elements of MKT: Common Content Knowledge (CCK), Specialised Content Knowledge (SCK), Horizon Content Knowledge (HCK), Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), and Knowledge of Content and the Curriculum (KCC). The organisation of the domains of knowledge is indicated in Figure 1 below. Definitions for each of the domains follow.

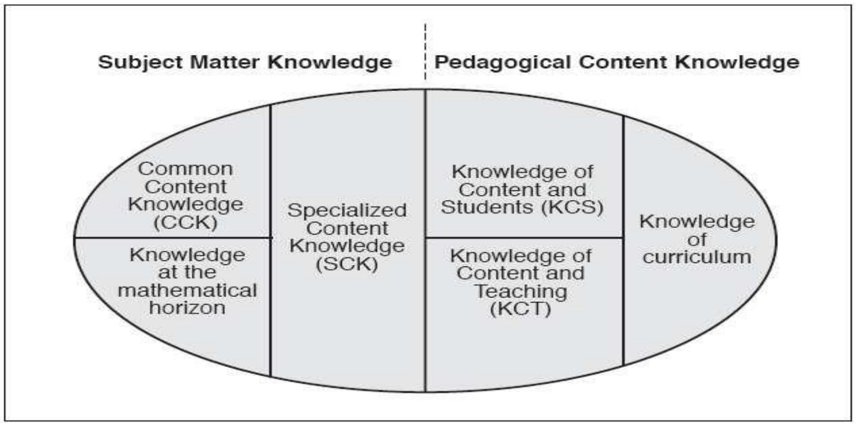

Figure 1: *Domains of Mathematical Knowledge for Teaching (Ball et al. 2008)*

Common content knowledge (CCK) is the mathematical knowledge held by an adult who can use a method to solve a mathematical problem (Ball et al., 2008). It involves correctly solving mathematical problems. Specialised content knowledge (SCK) is the mathematical knowledge and skill unique to teaching. It involves unique mathematical understanding and reasoning and requires knowledge beyond that being taught to learners. Knowledge of content and students (KCS) is knowledge that combines knowing about learners and knowing about mathematics. It involves anticipating what learners are likely to think and what they will find confusing as well as possible difficulties they may experience. It also involves recognising errors and identifying most likely errors learners make, interpreting learners' responses and developing learners' justifications. Knowledge of content and teaching (KCT) combines knowing about teaching and knowing about mathematics. Teachers have to have mathematical knowledge of the design of instruction. They must be able to sequence particular content for instruction and select and use effective instructional models. They have to identify what different methods and procedures to use to make the mathematics salient and usable by learners. It involves making decisions about what learner contributions to pursue and which to ignore or save for a later time.

While it may be clear that knowledge for teaching is multi-dimensional, the model categories appear static. It is somewhat difficult to decide where one category starts and where another finishes. However, this model tries to elaborate more specifically on what knowledge is required for teaching and to design suitable items to measure the different categories of knowledge. It recognises that definition and precision within each category is problematic.

Despite the static nature, the Ball et. al. (2008) model is a useful framework to begin to understand what mathematics teachers need to know for the task of teaching. I also find the mathematical knowledge for teaching framework to be useful when discussing different types of knowledge. As such, this framework informed this study. For this research, I studied preservice teachers' common content knowledge of equations, specialised content knowledge, knowledge of content and students and knowledge of content and teaching equations. I focused on the four MKT domains because I recognised that studying both SMK and PCK provides a much broader and deeper picture about students MKT than either can do alone. Furthermore, by trying to differentiate, distinguish and study the individual categories of MKT, we lose important information on how these categories interact with each other.

Equations

The second construct of the theoretical framework for this research largely draws from the work of Kriegler (2007). Kriegler developed a framework for algebraic thinking which is based on many years of work. Kriegler asserts that algebraic thinking is organised into two major components: the development of mathematical thinking tools and the study of fundamental algebraic ideas.

Mathematical thinking tools are analytical habits of mind. They are organised around three topics: problem solving skills, representation skills, and quantitative reasoning skills. Fundamental algebraic ideas represent the content domain in which mathematical thinking tools develop. They are explored through three lenses: algebra as generalised arithmetic, algebra as a language, and algebra as a tool for functions and mathematical modelling (Table 1).

Table 1: Components of Algebraic Thinking

Mathematical Thinking Tools	**Fundamental Algebraic Ideas**
Problem solving skills • Using problem solving strategies • Exploring multiple approaches/multiple solutions Representation skills • Displaying relationships visually, symbolically, numerically, verbally • Translating among different representations • Interpreting information within representations Quantitative reasoning skills • Analyzing problems to extract and quantify essential features • Inductive and deductive reasoning	Algebra as generalised arithmetic • Conceptually based computational strategies • Ratio and proportion • Estimation Algebra as the language of mathematics • Meaning of variables and variable expressions • Meaning of solutions • Understanding and using properties of the number system • Reading, writing, manipulating numbers and symbols using algebraic conventions • Using equivalent symbolic representations to manipulate formulas, expressions, equations, inequalities Algebra as a tool for functions and mathematical modelling • Seeking, expressing, generalizing patterns and rules in real-world contexts • Representing mathematical ideas using equations, tables, graphs, or words • Working with input/output patterns Developing coordinate graphing

The focus on algebraic thinking in this study involves problem solving, representation skills, quantitative reasoning skills, algebra as the language of mathematics and algebra as a tool for functions and mathematical modelling. This involves formulating and solving equations, describing relationships among different representations of equations, translating among different representations, interpreting information within representations, analysing problems to extract and quantify essential features, inductive and deductive reasoning. The algebraic thinking framework was used to analyse Mr. Mwati's common content knowledge.

Literature Review

Li's (2006) study characterises the mathematical knowledge upon which secondary school algebra teachers draw when pondering problem situations that could arise in the teaching and learning of solving algebraic equations. It also examines the potential connections between teachers' knowledge and their academic backgrounds and teaching experiences. Seventy-two middle school and high school algebra teachers in Texas participated in the study. Results revealed three topic areas in equation solving in which teachers' mathematical subject matter understanding should be strengthened: (a) the balancing method, (b) the concept of equivalent equations, and (c) the properties of linear equations in their general forms. The participants provided a wide range of instances of student misconceptions and difficulties in learning how to solve linear and quadratic equations, as well as a variety of strategies for helping students to improve their understanding. Teachers' subject matter knowledge played a central or prerequisite role in their reasoning and decision-making in specific contexts.

Li (2011) also describes the sequence of mathematical practices that a teacher designed and enacted during three consecutive lessons about four algebraic routines for solving quadratic equations, and focuses on the mathematical knowledge that is entailed in the teacher's actions and decisions. Among the three domains of mathematical knowledge for teaching (MKT), the teacher's knowledge of the equation solving routines played the most crucial role in shaping the lessons and potentially promoting mathematical proficiency in a balanced approach. The findings suggest that mathematics teacher preparation and professional development programs should provide more opportunities for teachers to revisit the features and applicability of various mathematical routines, and develop skills in making instructional decisions that would balance all domains of teachers' MKT, teacher beliefs, and other factors related to proficiency in algebraic routines.

In a different study of equations, Ellarton & Clements (2011) used a pencil-and-paper instrument to investigate prospective middle-school mathematics teachers' knowledge of equations and inequalities. Data analysis revealed that hardly any of the 328 students knew as much about elementary equations or inequalities as might reasonably have been expected.

Huang and Kulm (2012) also examined prospective middle grade mathematics teachers' knowledge of algebra for teaching with a focus on knowledge for teaching the concept of function. One hundred and fifteen prospective teachers from an interdisciplinary program for mathematics and science middle teacher preparation at a large public university in the USA participated in a survey. It was found that the participants had relatively limited knowledge of algebra for teaching. They also revealed weaknesses in selecting appropriate perspectives of the concept of function and flexibly using representations of quadratic functions. They made numerous mistakes in solving quadratic or irrational equations and in algebraic manipulation and reasoning. The participants' weaknesses in connecting algebraic and graphic representations resulted in their failure to solve quadratic inequalities and to judge the number of roots of quadratic functions. Follow-up interview further revealed the participants' lack of knowledge in solving problems by integrating algebraic and graphic representations.

My analysis of previous studies has found out that a lot of research has been carried out on teachers' mathematical knowledge for teaching (e.g. Ma, 1999; Hill et al., 2008; Bartell et al., 2012). However, these studies report more on elementary preservice and in-service teachers' mathematical knowledge for teaching rather than the knowledge of middle grades and high school preservice teachers. Not much research on preservice teachers' MKT of equations has been done in the Southern Africa Development Community (SADC) region and Malawi in particular. This study, therefore, is intended to fill this gap.

In addition, all these qualitative studies used tests, interviews and lesson recordings to investigate preservice teachers' mathematical knowledge for teaching. All the studies found out weaknesses in preservice teachers' MKT. The findings of these studies provide several implications for improving mathematics teacher preparation. For instance, Huang and Kulm (2012) recommend that at the teacher preparation curriculum level, it is important to provide an appropriate foundation for prospective teachers to obtain a sound and well-structured knowledge base needed for teaching. Mostly, they emphasise on deepening understanding of mathematics for preservice teachers as contrasted with connections of the mathematical topics to teaching. Ellarton & Clements (2011) propose a $5-R$ Intervention Model to help preservice teachers pass from darkness into light. They used this model in their study and found out that it placed many preservice teachers of middle school mathematics on a new track which they suggest would lead them into knowledgeable and competent teachers.

Research Methodology

According to Marshall and Rossman (1999), this research essentially falls into the qualitative design with multiple case study approach. Merriam (1998) asserts that qualitative research focuses on process, meaning and understanding. In order to arrive at judgments regarding preservice teachers' MKT, *"richly descriptive data"* were needed (Cohen and Manion 1980: 29). Words, therefore, were used instead of numbers to convey the outcomes of the research findings. In this study, I conducted semi-structured one-on-one interview (Patton, 2002). The interview was in two parts. The first part was a follow up of the paper and pencil tests that the participants wrote while the second part consisted of a task based-interview.

The tasks for the interview were selected basing on the fact that they fit in the theoretical framework. This means, the tasks assessed specific Ball's et. al. (2008) MKT domains. In addition, I also considered the dynamic nature of the MKT categories. For instance, an item would assess one category of MKT more strongly than the other. For example, it would assess CCK more than SCK, KCS and KCT or; in attempting to reveal his CCK, for example, Mr. Mwati would also reveal his KCS. Now, the selection of items to assess a particular MKT category was based on which category a particular item assessed more strongly than another. The tasks were then piloted prior to commencement of the formal data collection. The purpose of this pilot study was to assess the clarity of the problems on the students' paper and pencil tests and interview schedule as well as space and time issues relating to written students' tests and interviews and to check whether instructions were comprehensible and checking the validity and reliability of the results (Mayoux, 2013). Three preservice mathematics teachers participated in the pilot study.

Mr. Mwati was a final year Diploma in Education preservice teacher aged between 30 and 40. He was originally trained as a soldier. Later, he joined teaching and was trained as a primary school teacher for two years under the Integrated Primary Teacher Education program (IPTE). The IPTE program in Malawi is run in such a way that students learn theory for one year and they go for teaching practice in the other year before they graduate. After Mr. Mwati graduated as a primary school teacher, he taught at one of the primary schools in the Malawi defence force for two years before joining the secondary school teacher education program. He was the best student in his class.

I analysed data using thematic analysis (Ritche & Lewis, 2003; Creswell, 2008). Themes were developed from the theoretical framework as well as the data. For common content knowledge, I developed themes from the algebraic thinking model and the themes for the other MKT categories were developed from Ball's et. al. (2008) framework.

Results and Discussion

The analysis of Mr. Mwati's MKT shows that he demonstrated some evidence of conceptual understanding of equations. For instance, he was able to solve the equation $x^2 = 2x + 8$ by factor method, quadratic formula, completing squares and by graph in that order (see table 2). Being able to explore multiple approaches to a problem is an example of problem solving skills. Mr. Mwati also displayed deductive reasoning. When solving the equation $x^2 = 2x + 8$, Mr. Mwati used rules of algebra to solve for the unknown. Using rules of algebra to solve an equation is an indicator of deductive reasoning.

When I asked him which comes first between the quadratic formula and completing squares, Mr. Mwati stated that completing squares comes first because the quadratic formula is derived from the general quadratic equation by completing squares. Despite that he solved the equation using the quadratic formula first before using completing the square method, Mr. Mwati knew that completing the square is the prerequisite for the quadratic formula.

Coming up with a table and graph from an equation may imply that Mr. Mwati was also able to display relationships visually and to translate among different representations. It might also be inferred that Mr. Mwati was able to represent mathematical ideas using equations, tables and graphs. Additionally, he interpreted the graph that I gave him before he answered the questions related to the graph. Ability to interpret a graph indicates that Mr. Mwati was able to interpret information within a representation. The problem solving skills that he displayed enabled him to interpret the graph that he was given during the interview.

Table 2: Common Content Knowledge

Preservice Teacher's Knowledge (CCK)	Examples from Mr. Mwati
Recognising why a particular method failed and what to do	**Mwati**: Aaah! It is because we are looking at factors of -10 which could give us the coefficient of x. Now, the factors of -10 that could give us the coefficient of x, we don't have such. That is why I chose to use the other method for solving the particular equation. **Mwati:** … Because factor method involves factors. There are some problems in which you can have factors but when you add or subtract them, you cannot get the coefficient of x. Now, if that situation comes in, it is when we are saying we need to have other methods of solving such equations because factorisation now failed…
Exploring multiple approaches to a problem	**Mwati**: Yes, because this is factorisation, I will also use quadratic formula to solve the equation **Mwati**: I think the problem can also be solved by graphical method. … We can also use completing the square.
Displaying relationships visually / Translating among different representations	**Mwati**: A method I can call the final one is graphical method. **FM**: ok. Can you now solve the equation graphically? **Mwati:** We need to equate this equation to y. Now, it will be $x^2-2x-8=y$. Now from there we need to find the range of values of x. Maybe, we can start from -3 to 5. This range has just been chosen arbitrarily. Now we need to have a table whereby we are saying the values of x should range from -3 to 5.

Although Mr. Mwati displayed some evidence of conceptual understanding; we cannot assume that he has a comprehensive and well-articulated mathematical knowledge for teaching. There are some things that he needs to improve. For example, when changing a quadratic equation to standard form, he referred to this process as equating the equation to zero. The more precise way of putting this is to refer to the process as to equate all terms to zero or just to change the quadratic equation to standard form. Secondly, Mr. Mwati did not pronounce the "coefficient of x^2". Instead, he pronounced it as the coefficient of the highest power. Furthermore, the change from $x^2=2x+8$ to $x^2-2x-8=0$ was unnecessary when solving the quadratic equation by completing the square. A solution process which begins as this one indicates the participant's reliance on procedural knowledge and some overgeneralisation of rules. An example of such an overgeneralisation of a rule when solving quadratic equations is "when we solve quadratic equations we are required to change them to standard form". As much as this works when solving by factor method or the quadratic formula, the rule does not apply to equations such as $x^2=2x+8$ when we want to solve by completing the square. Mr. Mwati also had difficulties with posing a word problem that was meaningful and correct for the given symbolic problem $2x+4=3x-9$. He pronounced $2x$ as multiplying x twice and $3x$ as multiplying x three times. The language that he used is not conventional. According to Adler and Patahuddin (2012), mispronouncing terms is an indication of limited common content knowledge.

For specialised content knowledge, the interview tasks involved explanations, representations, working with learners' ideas and questioning (see Kazima, Pillay, & Adler: 2008). The specialised content knowledge that Mr. Mwati demonstrated during the interview includes giving mathematical explanations, linking representations to underlying ideas and connecting a topic being taught to topics from prior or future years. When I asked him why he added the square of half of the coefficient of x to both sides of the equation when solving the equation by completing the square, Mr. Mwati explained that he did that in order to make the left hand side of the equation a perfect square so that in the end he could factorise. Of the many descriptors of SCK described by Kazima et al (2008), only four descriptors were identified during the interview with Mr. Mwati (see table 3). One possible cause might be that the questions posed were more suitable for getting access to the participants' CCK. It could also be because in Ball's et. al. (2008) framework, there is no clear distinction between CCK and SCK. The results may also reflect the fact that specialised content knowledge is the knowledge enacted in practice hence not much of the specialised content knowledge was displayed during the interviews. Similar results were found from a study conducted by Huang (2012) in the USA and China. It was found out that the participants had relatively limited knowledge of algebra for teaching. They also revealed weaknesses in selecting appropriate perspectives of the concept of function and using representations of quadratic functions flexibly.

Table 3: Specialised Content Knowledge

Preservice Teacher's Knowledge (SCK)	**Examples from Mr. Mwati**
Linking representations to underlying ideas	**FM:** .... Now, what is the relationship between the number of roots and the graph of a quadratic equation? **Mwati:** Ok. The relationship is that the graph is cutting at the values of x two times like in the graph I drew.
Connecting a topic being taught to topics from prior or future years	**Mwati:** (To solve this problem), I think they need to have the knowledge of the Cartesian plane where we have the x axis and the y axis and its values. Now, if they know the values of x they are going to tackle this problem correctly.
Being able to show what steps of a procedure mean and why they make sense	**FM:** Why should you add to both sides the square of half of the coefficient of x? **Mwati:** We want to make this a perfect square because we are completing the square. We are adding half of the coefficient of x to make it a perfect square. **FM:** … (Relationship between roots and graph of quadratic equation). Why do you think this relationship exists? **Mwati**: Because the highest power of the equation is 2. That is why we are having the graph cutting the x axis in 2 places.

For knowledge of content and students, Mr Mwati was able to anticipate students' thinking and confusions, identify students' misconceptions, understand reasons for the misconceptions and ask questions to understand or reveal students' reasoning and misconceptions. However, Mr. Mwati displayed limited ability to ask questions to understand or reveal students' reasoning and misconceptions. This was indicated by the fact that he came up with one question only when I asked him to give examples of questions he could ask in order to identify students' misconceptions. The findings also reveal that Mr. Mwati was able to identify the source of students' misconceptions, and errors but had difficulty in generating effective ways different from telling the rules or procedures to confront such misconceptions. Although there are a number of more conceptual approaches to address students' difficulties, errors and misconceptions, Mr. Mwati did not mention them during the interviews (table 4). Kılıç (2011) also found similar results when the researcher studied preservice secondary mathematics teachers' knowledge of students.

Table 4: Knowledge of Content and students

Preservice Teacher's Knowledge (KCS)	Examples from Mr. Mwati
Anticipating student thinking and confusions	**FM**: Now what errors may students exhibit as they try to answer this question? **Mwati:** Maybe to find the correct ^{0}F if the graph is cutting were there are no numbers. **FM**: What do you mean when you say "where there are no numbers"? **Mwati:** The numbers between 0 and 10 or between 10 and 20. Now because the numbers are not written here, the students will confuse.
Identifying students' misconceptions	**FM:** Now, what misconceptions might lead to the error you have mentioned? **Mwati:** Maybe they may be thinking that these are the only degrees; we do not have other degrees apart from the ones we are being given here.
Understanding reasons for the misconceptions	**FM:** How would that thinking develop? **Mwati:** It will develop due to the way they are taught by their teacher maybe the teacher did not explain that in between we have numbers. Now if the teacher did not explain this, it can be a source of the errors or the misconceptions
Asking questions to understand or reveal students' reasoning and misconceptions	**FM:** Now suppose you want to teach this to your students, what questions may you ask your students to identify the misconceptions? **Mwati**: Maybe we can ask for the numbers that are not written on the graph. For example, how many degrees Fahrenheit are equivalent to 93^0 C? The answer that the students will give will show whether they have misconceptions or not.

Just as with specialised content knowledge and knowledge of content and students, Mr. Mwati displayed limited knowledge of content and teaching. For instance, in the case of interpreting a linear graph, Mr. Mwati stated that he would explain and demonstrate to the students that there are numbers in between say 10 and 20, 20 and 30, although they were not written on the x axis of the graph. When it came to how he would present the equation $x^2 = 2x + 8$, Mr. Mwati explained that he would give them steps to follow for them to come up with the roots of the equation. Mostly, Mr. Mwati relied on explanation and demonstration as methods of enhancing students' conceptual understanding. Analysis has shown that he mentioned only one case where he could ask questions to students or ask volunteers to explain their solution processes (see table 5). These findings illuminate results of a study conducted by Li (2011) who found out that one preservice teacher's mathematical knowledge for teaching algebraic routines was weak.

Table 5: Knowledge of Content and Teaching

Preservice Teacher's Knowledge (KCT)	Examples from Mr. Mwati
Knowledge of representations for teaching particular topics	**Mwati**: … We can represent that in an equation. For example, to say maybe, you are taking age between father and son. Maybe the father's age can be twice the age of the son. This can be represented as an equation
Explanation and demonstration as a means of enhancing conceptual understanding	**Mwati**: I would explain to them that there are numbers in between say 10 and 20, 20 and 30 etc although they were not written on the graph. **Mwati:** I would use demonstration and explanation.
Knowledge of instructional strategies	**Mwati**: I will give them steps to follow for them to come up with the roots of the equation. **Mwati**: Giving them questions and asking volunteers to come in front to give the solution in any way they want so that I can say ok this is the way the students understood the problem.

Despite the high success rate of solving the equation given during the interview session, Mr. Mwati displayed some misconceptions which were evident during the analysis. For instance, Mr. Mwati regarded variable as object (see table 6).

Table 6: Misconceptions

Preservice Teacher's Misconception	Example from Mr. Mwati
Variable as object	**FM**: How could you let learners understand that x and y could represent anything? **Mwati**: Ok. We can take for example things around them. For example; mangoes, textbooks people or even learners themselves. We can call and let them be like letters like x and y representing an object or any material

When I asked him how he could deal with a misconception about variable as object, Mr. Mwati stated that he could deal with a student's misconception by letting x and y represent anything. Mr. Mwati displayed this misconception during a follow up interview of the test results. This is how the conversation went:

FM: Let us look at question 2b. In this question, you are saying you could let learners understand that x and y can represent anything. How could you let learners understand that x and y could represent anything?

> *Mwati: We can take for example things around them, for example; mangoes, textbooks, people or even learners themselves. We can call and let them be like letters like x and y representing an object or any material.*

Mr. Mwati's interpretation of variable as object violates the mathematical meaning of variable which represents a number. In addition to interpreting variable as object, Mr. Mwati did not link representations to underlying ideas and to other representations. He also had difficulties with posing a word problem that was meaningful and correct for the given symbolic problem $2x+4=3x-9$.

Conclusion

In this study, Mr. Mwati's mathematical knowledge for teaching equations was investigated through a one-on-one interview. On one hand, Mr. Mwati displayed success when solving the equation $x^2=2x+8$ using four methods namely, factoring, completing the square, quadratic formula and by graph. When solving the equation $x^2=2x+8$, he was able to explore multiple approaches to a problem, to use rules of algebra to solve an equation and hence displayed deductive reasoning, to display relationships visually and to translate among different representations and to interpret information within a representation. All these are algebraic thinking skills conceptualised by Kriegler (2012) in the theoretical framework. On the other hand, Mr. Mwati displayed some misconceptions and other difficulties. The results have also shown that Mr. Mwati had difficulties with pronouncing terms. He also unnecessarily changed the equation $x^2=2x+8$ to standard form when solving by completing the square. Mr. Mwati also had difficulties in asking effective questions to identify students' errors and misconceptions. Similarly, Mr. Mwati's specialised content knowledge and knowledge of content and teaching were limited.

The fact that Mr. Mwati displayed limited MKT during the interview could also result from the fact that the measurement of his knowledge within the categories of SCK, KCS and KCT was somewhat constrained due to the limitation of not having access to classroom students. In his responses, Mr. Mwati had to 'work' within a hypothetical situation. While his CCK was more evident during the task-based interview, it is possible that given a wider range of problems, additional evidence of knowledge within CCK, SCK, KCS and KCT categories could be obtained. It is also possible that additional evidence of MKT would be obtained using tests and video lesson recordings.

Finally, the findings in this study suggest that Mr. Mwati lacks the knowledge necessary for teaching equations effectively to his students. Since this apparent lack of knowledge could be observed with one of the best students, it appears that Malawian teacher education needs to emphasise on developing preservice teachers' MKT. This aspect is lacking in contemporary education of secondary school mathematics teachers in Malawi. With sufficient mathematical knowledge for teaching, they will be able to interpret student thinking, identify misconceptions provide tasks and pose questions that will guide students' interpretations of mathematics. The teachers will also be able to find appropriate strategies for inducing cognitive conflict that will help students to deconstruct their naïve theories and reconstruct correct mathematical conceptions. The results also suggest that the implications for translating CCK into effective SCK, KCS and KCT are paramount in raising the profile of mathematics teaching and learning.

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