# A preservice teacher's choice and use of examples in teaching exponential equations

Florence Mamba

*University of Malawi*

## Abstract

*This study investigated a preservice secondary school teacher's choice and use of examples in the teaching of exponential equations. The study was carried out with one preservice secondary school teacher. Data were generated using video lesson recordings and lesson plans and analysed using thematic analysis. By analysing and characterising the preservice teacher's choice and use of examples, I was able to identify significant aspects of his mathematical knowledge base that might have supported or limited students' learning. The implications of these findings on mathematics teacher preparation are discussed.*

**Key Words**: *Examples, preservice teacher, secondary school, exponential equations*

## Introduction

Research indicates that examples are central to mathematical thinking, learning and teaching, particularly with respect to conceptualisation, generalisation, abstraction, argumentation, and analytical thinking (Zodik & Zaslavsky, 2008). Examples are a particular case of a larger class, from which students can reason and generalise (Watson & Mason's, 2002). Examples offer insight into the nature of mathematics through their use to demonstrate methods, to indicate relationships, and in explanations and proofs. Examples are a communication device that is fundamental to explanations and mathematical discourse (Bills et al., 2006). In order to improve students' performance in mathematics in general, the teacher should be able to "choose which examples to start with and which examples to use to take students deeper into the content" (Ball, Thames, & Phelps, 2008, p. 401). In addition, teachers need to choose the most powerful analogies, illustrations, examples, explanations and find out ways of representing the subject which makes it comprehensible to others (Shulman, 1986). Despite this call by researchers, Malawian students perform poorly in national examinations (Malawi National Examinations Board, 2008–2013). Analyses of Malawi National Examinations Chief Examiners' Reports for years 2008 to 2013 indicate students' poor performance in Algebra in general and equations in particular. The results indicate students' inability to formulate and solve equations. When solving exponential equations, students could not change the equations to quadratic form and produced incomplete solution processes (Malawi National Examinations Board, 2008–2013). Research reports on choice and use of examples among Malawian preservice secondary school mathematics teachers are also less available. Hence the aim of this study was to investigate a preservice secondary school teacher's choice and use of examples in the teaching of exponential equations. The study would inform preservice secondary school teacher educators about the content of mathematics teacher preparation.

## Exponential equations

Exponential equations are equations in which the unknown occurs in the exponent. There are different types of exponential equations found in the secondary school curriculum in Malawi (Malawi Government, 2000). Firstly, there are exponential equations in which the unknown occurs just once and that each side of the equation can be expressed in terms of the same base. Such type of equations can be solved using the property that if the bases are the same, the exponents are equal. Secondly, exponential equations in which the unknown occurs just once and that each side cannot be expressed to the same base are solved using logarithms. The third type includes exponential equations which contain exponentials such as $b^x$, $b^{2x}$ and $b^{2x+1}$. Such exponential equations are solved using quadratic equations. In Malawi, exponential equations are introduced to students in grade 11 after students' exposure to indices, logarithms, laws of indices and laws of logarithms in grade 10. In a study reported in this paper, the preservice teacher chose and used examples of exponential equations which can be expressed in terms of the same base and those that contain exponentials such as $b^x$, $b^{2x}$ and $b^{2x+1}$.

## Literature review

Various authors have categorised examples in different ways. For example, Rissland-Michener (1978) distinguishes four types of examples (not necessarily disjoint), which have epistemological significance. These include; Start-up examples (motivation for and initiation into a topic), reference examples (basic, widely applicable, standard cases), model examples (paradigmatic, generic examples) and counter-examples (demonstrating that conjectures are false). Rowland, Thwaites, and Huckstep (2003) distinguish between providing examples of something as raw material for inductive reasoning, as particular instances of generality, and secondly, providing an environment for practice. The authors recognise that practice exercises might also lead to different kinds of awareness and comprehension.

Bills et al. (2006) distinguish between examples of a concept and examples of the application of a procedure. Those examples which demonstrate procedures are further classified into worked-out examples, in which the procedure is performed by the teacher or text-book author often with some sort of explanation; or commentary exercises, which consist of tasks set out for students to complete. Zaslavsky (2010) also categorises examples into specific, semi-general and general examples, according to their "explanatory power". She identifies the following uses teacher's examples: conveying generality and invariance, explaining and justifying notations and conventions, establishing the status of pupil's conjectures and assertions, connecting mathematical concepts to real life experiences, and the challenge of constructing examples with given constraints. Zaslavsky concludes that, "choosing instructional examples entails many complex and even competing considerations, some of which can be made in advance, and that others come up during the actual teaching" (p. 126).

Since examples are chosen from a range of possibilities (Watson & Mason, 2002), teachers need to recognise that some examples are "better" than others (Rowland et al., 2003). A good example is one which is transparent to the students, helpful in clarifying and resolving mathematical subtleties and generalisable (Bills et al., 2006). Bills and colleagues maintain that the specific representation of an example or set of examples and the respective focus of attention facilitated by the teacher, have bearing on what students notice, and consequently on their mathematical understanding. Inappropriate examples can lead to construction of understanding that was not the intention of the teacher. Thus examples need to be introduced carefully, to support the distinction between critical and non-critical features and the construction of rich and appropriate concept images and example spaces (Vinner, 1982; Watson & Mason, 2002).

Although exemplification is highly demanding, it has not been extensively investigated with regard to the teacher (Zaslavsky, 2010; Bills et al., 2006). Ball, Thames and Phelps (2008) also raise concerns regarding the knowledge base teachers need in order to carefully select appropriate examples that are useful for highlighting salient mathematical issues.

## Theoretical Framework

According to Zodik and Zaslavsky (2008), three aspects of teacher knowledge strongly relate to exemplification in mathematics education. These include knowledge of mathematics, knowledge of students' learning, and knowledge of pedagogy. Thus the theoretical perspective for this study is based on the mathematical knowledge for teaching model which was developed by Ball et al. (2008). The mathematical knowledge for teaching (MKT) model elaborates the use of the Shulman's (1986) subject matter knowledge, pedagogical content knowledge and curriculum knowledge categories, organising and defining them in a different way. There are six elements of MKT: common content knowledge (CCK), specialised content knowledge (SCK), horizon content knowledge (HCK), knowledge of content and teaching (KCT), knowledge of content and students (KCS), and knowledge of content and the curriculum (KCC). The organisation of the domains of knowledge is indicated in Figure 1.

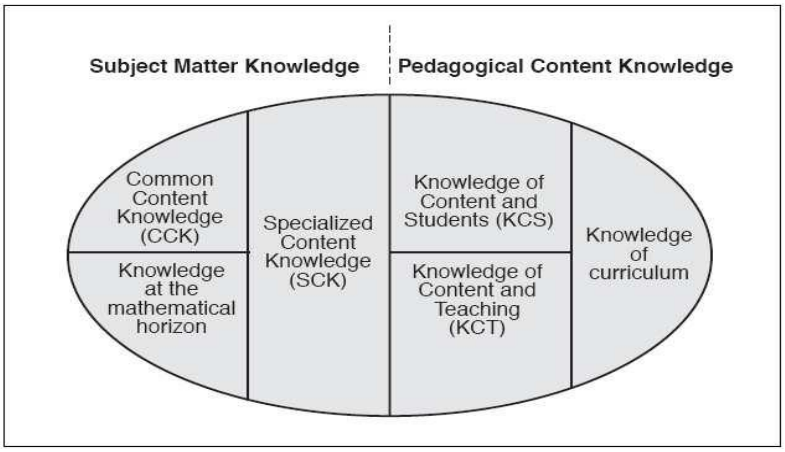



Figure 1: *Domains of Mathematical Knowledge for Teaching (Ball et al., 2008)*

CCK is mathematical knowledge and skill used in general settings, settings not necessarily unique to teaching. It involves calculating an answer correctly, solving mathematical problems correctly, understanding the mathematics, recognising when a student gives a wrong answer and recognising when a text book is inaccurate. SCK is mathematical knowledge and skill particular to teaching. It involves connecting a topic to topics from prior years, designing mathematically accurate explanations that are comprehensible and useful for students, making features of a problem visible to and learnable by students and posing mathematical questions and problems that are productive for students' learning. KCS is knowledge which is a combination of knowing about students and about mathematics; anticipating students’ errors and misconceptions, identifying students’ errors and misconceptions and asking questions to real or understand students’ errors and misconceptions. KCT is a combination of knowing about teaching and about mathematics. It involves finding appropriate mathematical examples, choosing which examples to start with and which examples to use to take students deeper into the content, modifying tasks to be either easier or harder, deciding when to pose a new task to further students’ learning and evaluating instructional strategies.

## Research Method

This research falls into the qualitative descriptive case study design. Merriam (2009) asserts that qualitative research focuses on process, meaning and understanding. In order to arrive at judgments regarding preservice teachers’ choice and use of examples, “richly descriptive data” were needed (Merriam, 2009, p. 14). Words, therefore, were used instead of numbers to convey the outcomes of the research findings.

The study reported here is part of a large study that investigates four preservice secondary school teachers’ mathematical knowledge for teaching equations. Part of this study was also presented at the 24$^{th}$ conference for Southern African Association for Research in Mathematics and Technology Education (SAARMSTE) (Mamba, 2016a) and the 40$^{th}$ conference for the international group of the Psychology of Mathematics Education (Mamba, 2016b). In the papers, I reported about results from a task based interview that I conducted with one preservice secondary school mathematics teacher, Mwati (pseudonym) and video lesson observation for one lesson. In the current paper, I explore how Mwati chose and used examples in teaching of exponential equations. Mwati was in his final year of study at the time the data was generated. He was the best student in his class. I thought the best student was the best participant to choose because he was an “information-rich” case for in-depth study (Patton, 2002). He was originally trained as a primary school teacher and had taught in primary school for two years before joining the secondary school teacher education programme.

The lesson was for grade 11 (upper secondary school). It was about solving exponential equations. In the final semester of the final year of study, students at Mwati’s institution are given an opportunity to practise teaching at a secondary school that is attached to the college. This teaching was, hence, part of Mwati’s teacher training programme. I generated data using video lesson recordings. I also collected lesson plans for all the lessons. Using video allowed me to record the events thoroughly. It also allowed me to look at the episode multiple times for further analysis (Girden & Kabacoff, 2011). Before beginning the data generation, I asked for permission from the principal of the concerned college, the school head teacher and the mathematics teacher of the concerned class.

When recording the video, I used a single camera and made continuous recordings with panning and zooming carefully. I used a wide camera to capture the whole scene. The camera was fixed on a tripod high and in the corner of the classroom. I started the camera before the lesson began. I recorded continuously and continued until the teacher left the classroom.

I analysed the data using thematic analysis. Firstly, I transcribed the video recordings in whole. Then, I analysed transcripts of selected episodes. I chose the particular episodes because some elements that Mwati displayed in these episodes were unusual for the Malawi context. I developed themes both *a priori* from the theoretical framework and *a posteriori* from the data (Powell, Francisco, & Maher, 2003). Analysis for the transcripts and lesson plans involved identification of the mathematical knowledge that came into play as Mwati chose and used the examples. To achieve credibility of the results, another researcher analysed the data. We agreed in at least 90% of all the themes, with no discussion between the researchers. Furthermore, the findings were read and critiqued by other researchers.

## Results and Discussion

### Mwati's choice of examples

The following were the examples that Mwati chose:

Lesson 1: Solve $81^n = 3$, $4^{x+1} = 32$, $3^{2x-5} = 27$, $8^{x-1} = 32^x$

Lesson 2: Solve $x^2 = 16$, $9^{(a-3)} \times 81^{(1-a)} = 27^{-a}$, $9^x \times 3^{(2x-1)} = 3^{15}$, $8 \times 2^{2x-1} = 16 \times 2^{x-1}$ and $\frac{3^{4x}}{9^x} = 729$

Lesson 3: Solve $2^{ab} - 9 \times 2^b + 8 = 0$, $5^{2x} - 6 \times 5^x + 5 = 0$, $3^{2n} - 6 \times 3^n + 9 = 0$ and $2^{2b+1} - 17 \times 2^b + 8 = 0$

### Mwati's use of examples

**Lesson summary**

The lessons were for grade 11. They were about solving exponential equations.

**Lesson 1**

Mwati started by asking students to express 8 in base 2, 27 in base 3 and 32 in base 2. Thereafter, he asked students to solve $81^n = 3$. He put on the chalkboard a chart with 7 laws of indices and asked students to refer to it when solving the equation. Before students attempted the example, Mwati asked them to solve the equation together. He solved the equation by equating exponents. Then Mwati gave the students $4^{x+1} = 32$ to solve individually as an exercise. After students had solved the equation, he invited some to solve the equation on the chalkboard.

**Lesson 2**

Mwati began by asking students to solve. $x^2 = 16$. The difference between this example and those for the first lesson was that in this example, the unknown was now the base. Then he asked them to attempt solving $9^{(a-3)} \times 81^{(1-a)} = 27^{-a}$. There was lack of continuity between the first example and the other examples because in the second and the other examples, the unknown was the exponent just like in the first lesson. After he saw that students were struggling to come up with the solution process, he asked them do solve the equation together with him on the chalkboard. Then he gave them three exercise examples; $9^x \times 3^{(2x-1)} = 3^{15}$, $8 \times 2^{2x-1} = 16 \times 2^{x-1}$ and $\frac{3^{4x}}{9^x} = 729$ which they did not finish because time was over. He asked them to do the remaining work as home work. In all the examples, Mwati encouraged the students to use the seven laws of indices he had brought for them in the previous lesson.

**Lesson 3**

Mwati started by asking students to evaluate $3^b \times 3^b$, $5^{2a} \times 5^1$ and $2^{x+1}$. Students were required to use laws of indices to work out these problems. Then, Mwati solved the equation $2^{2b} - 9 \times 2^b + 8 = 0$ which was discussed throughout the lesson. Mwati linked exponential equations to quadratic equations at a high level of abstraction. He related the exponential equation to the quadratic equation $x^2 - 9x + 8 = 0$ and continued with solving the two parallel equations. Then, Mwati gave the students the equation $5^{2x} - 6(5^x) + 5 = 0$ to solve in pairs. Few pairs solved the equation and one pair presented their work to the whole group. Finally, Mwati gave the students $3^{2n} - 6(3^n) + 9 = 0$ and $2^{2b+1} - 17(2^b) + 8 = 0$ to do as homework. The way Mwati solved the equation in this lesson was unusual in Malawi context. An extract of this work is presented in Transcript 1.

## Findings

### Preservice teacher's choice and use of examples

Mwati's choice of the examples depended much on his knowledge of students and his knowledge of content and teaching. For instance, his choice of the examples depended much on his knowledge of methods of solving exponential equations. Findings reveal that he chose appropriate mathematical examples and that the examples were in order of complexity. In addition, findings reveal that some examples were pre-planned while others were spontaneous. Analysis also shows that Mwati's use of the examples was dependent on his common content knowledge, specialised content knowledge, knowledge of content and students and knowledge of content and teaching. Thus the way he used the examples displayed his knowledge of methods of solving exponential equations, his explanations, questioning techniques and his knowledge of students' thinking.

Analysis of Mwati's choice of examples involved examining the extent to which the examples suited the object of learning. The object of learning in this case was solving exponential equations. All examples suited the object of learning and were correct. "Correct" here means freedom from error (Soanes, Hawker, & Elliot, 2006). Thus the examples that Mwati chose were appropriate for exemplifying methods of solving exponential equations.

These examples are in the order they were chosen and presented to students. They were used to generalise the methods of solving exponential equations. They ranged from simple to complex and were solved by equating exponents and reducing to quadratic equations. The examples also relate to what Zazkis and Leikin (2007) call richness. In this sense, the examples varied in kind. However, they were routine examples. Mwati's choice of the examples also relates to accessibility and correctness. For Mwati, these examples were selected from a textbook hence there was no struggle in finding the examples. During selection, Mwati ignored the examples that were incorrect but brought to the classroom only examples that he was sure were correct. The examples were specific but represented the generality of methods of solving exponential equations.

The way Mwati chose and used the examples clearly displays knowledge of the mathematics that he was teaching. In the first lesson, he put up the main idea on the chalkboard and explained it to the students. The main idea was solving exponential equations. During the subsequent lessons, methods of solving exponential equations were evident. It is clear that Mwati's choice of examples to teach the methods of solving exponential equations was based on his knowledge of the nature of exponential equations; equations solved by equating exponents and equations of the quadratic form. During the lessons, Mwati could recognise students' incorrect answers and text book errors. He also solved mathematical problems correctly.

Although the methods of solving exponential equations were evident in the examples, Mwati did not make them explicit during the lessons. He did not draw students' attention to the different forms of the exponential equations being solved. Moreover, in all the three lessons, he did not explain to the students why he was using the different examples and the different methods of solution. Mwati did also not draw students' attention to the nature of solutions for the different examples which were discussed. The main focus for Mwati was the procedure for solving each equation. Drawing students' attention to the different forms of the exponential equations being solved, the methods used and the nature of their solutions, was necessary because it could help make the critical features transparent to the students that is, allowing the students to see the general through the particular (Zaslavsky, 2010).

**Explanation**

"…Examples are fundamental elements of an explanation" (Zaslavsky, 2010, p. 107 ). In this study, any attempt to use the example to provide understanding was regarded as an explanation. During the teaching and learning process, Mwati gave several explanations to enhance students' understanding of the methods of solving exponential equations. Some explanations were comprehensible and useful for students unlike others. Firstly, Mwati explained that in an exponential equation, when the bases are the same, the powers are equal. In such explanations, Mwati was able to connect exponential equations to linear equations. It might, therefore, be inferred that Mwati was able to connect a topic to prior years. Another case where he displayed ability to connect a topic to prior years was when he reduced an exponential equation to a quadratic one. When explaining about the use of quadratic equations to solve $2^{2b} - 9 \times 2^{b} + 8 = 0$, he stated that $2^{b}$ should be regarded as the unknown. Transcript 1 provides explanations that Mwati gave about solving exponential equations by reducing them to quadratic equations.

**Transcript 1**

T: Let us consider the equation $2^{2b} - 9 \times 2^{b} + 8 = 0$ and the term $2^{2b}$. Let us break it like the way we did with $3^{b} \times 3^{b} = 3^{2b}$. So what will it be equal to?

Ralph: It will be $2^{2b} = 2^{b} \times 2^{b}$.

T: It will be $2^{2b} = 2^{b} \times 2^{b}$ or $2^{2b} = (2^{b})^{2}$. Does that make sense?

Students: Yes.

T: Ok. Let us look at the laws of indices.

[He brought the chart with the laws of indices that he used on first and second day.]

T: Law number 3. They are saying $\left(x^{a}\right)^{b} = x^{ab}$. Like in the problem we are solving $2^{2b} = \left(2^{b}\right)^{2}$. Therefore, the equation will be $\left(2^{b}\right)^{2} - 9\left(2^{b}\right) + 8 = 0$ Now, from there, do you remember having done quadratic equations?

Students: Yes.

T: What we did on quadratic equations can be applied here. So we can make the equation be $x^{2} - 9x + 8 = 0$. Have you seen that?

Students: Yes.

T: Therefore, this $2^{b}$ has to be taken as unknown. Therefore, we can solve it as a quadratic equation because this $2^{b}$ has been taken as unknown. Now, let us proceed working on $\left(2^{b}\right)^{2} - 9\left(2^{b}\right) + 8 = 0$. What can we do from here? If you are given an equation like $x^{2} - 9x + 8 = 0$. What is the next step here?

Rose: We need to factorise.

T: How can we factorise this one?

Charles: We can factorise by $x^{2} - 8x - x + 8 = 0$.

T: Yes. Borrowing this information, can you apply it on $\left(2^{b}\right)^{2} - 9\left(2^{b}\right) + 8 = 0$?

George: $\left(2^{b}\right)^{2} - 8\left(2^{b}\right) - 2^{b} + 8 = 0$.

T: Do we all agree?

Students: Yes (some), No (others).

T: Let us agree. I was saying we need to apply the same knowledge we know about $x^{2} - 8x - x + 8 = 0$ and apply that knowledge to $\left(2^{b}\right)^{2} - 9\left(2^{b}\right) + 8 = 0$. Now this $2^{b}$ has to be taken as unknown. Now from here, we can factorise the equation because we have $2^{2b} = 2^{b} \times 2^{b}$. So, we have $2^{b}\left(2^{b} - 8\right) - 1\left(2^{b} - 8\right) = 0$. Have you seen that?

Students: (Complaining, murmuring).

T: Look at $x^{2} - 8x - x + 8 = 0$. Factor out $x$ to have $x\left(x - 8\right) - 1\left(x - 8\right) = 0$.

[The teacher solved two parallel equations.]

Students: We are confused.

T: This ( $x(x-8)-1(x-8)=0$ ) is more or less like ( $2^b(2^b-8)-1(2^b-8)=0$ ).

Yohave: I did not understand the second step. Where has the 9 gone?

T: At the second stage, where is the 9? [The teacher used $x^2-8x-x+8=0$ ].

Future: $-8x-x$.

T: Here in $(2^b)^2-8(2^b)-2^b+8=0$, the 9 has been substituted by $-8(2^b)-2^b$. That means if we add or subtract $-8(2^b)-2^b$ we will get $-9(2^b)$. Have you seen that?

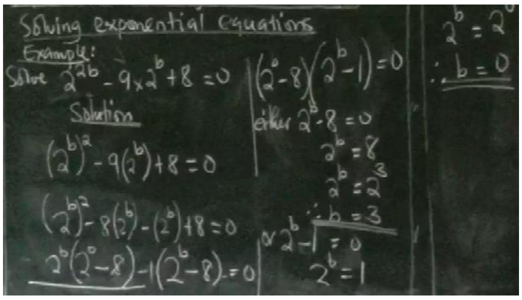
Students: Yes (some), (Others did not answer).

T: Ok. Let us proceed. Now from $x(x-8)-1(x-8)=0$, we need to factor out the bracket $(2^b-8)$. This gives us $(2^b-8)(2^b-1)=0$. This is the same as in $x^2-8x-x+8=0$. In this one, we have $(x-8)(x-1)=0$. So, we have either $x-8=0$ or $x-1=0$. Isn't it? Now, taking the same knowledge to our exponential equation, we have ether $2^b-8=0$ or $2^b-1=0$.

Mwati continued solving the equations $2^b-8=0$ and $2^b-1=0$ until he found that either $b=3$ or $b=0$.

Mwati's solution processes are presented in figure 2.

A

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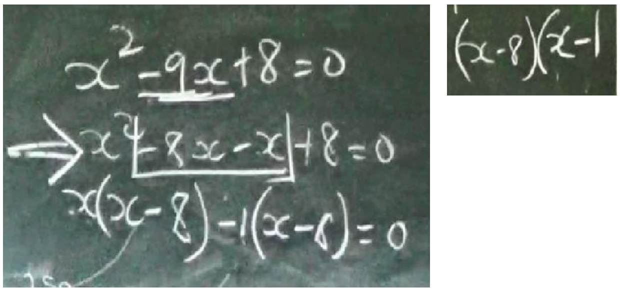


*Figure 2: Mwati's parallel solution processes for* $2^{2b}-9\times 2^b+8=0$

Mwati did not finish solving the parallel example. During his presentation of the example, the students seemed to follow how the teacher solved the two equations but they got lost in the middle and they explained to Mwati that they were confused. Many students expressed that they did not understand. Some of them had questions but the teacher ordered them to reserve those questions and solve the equation $5^{2x}-6\times 5^x+5=0$ as an exercise. After some minutes, Mwati asked the students to present their solution processes. Maria solved $5^{2x}-6\times 5^x+5=0$ as follows:

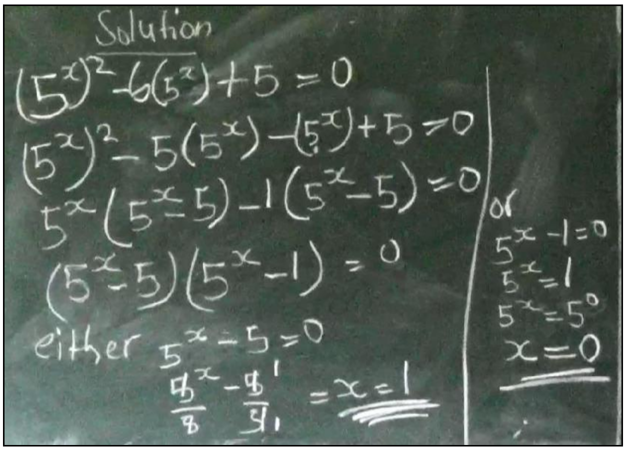


*Figure 3: Student's solution process for* $5^{2x} - 6 \times 5^x + 5 = 0$

In figure 3, Mwati noted that at the end of the solution process, Maria divided the base by itself of which she confused with ignoring the bases when the exponents are the same. Mwati erased the student's work and wrote $5^x = 5 \Rightarrow 5^x = 5^1$. He concluded that since the bases are the same, the exponents must be equal.

It would seem that Mwati presented the lesson at a high level of abstraction and he did not appear to be able to explain why the methods worked. In addition, Mwati's presentation of the solution process to the equation differed from his plan which indicated that he would let $x = 2^b$ so as to convert the equation to $x^2 - 9x + 8 = 0$. As such, he did not make the relevant features of the example visible and learnable by most students hence almost 90% of the students were unable to solve the given exponential equation.

**Questioning**

Analysis of the questions that Mwati asked, regarding the examples he gave, was based on Bloom's taxonomy (Krathwohl, 2002). Transcript 1 shows that Mwati asked factual and low order thinking questions. Other questions required 'yes' or 'no' answers. In some cases, Mwati asked two or three questions at the same time. In such cases, students answered the last one and the teacher did not bother following up the answers to the other questions. He could also ignore students' responses even when he gave them an opportunity to answer questions individually. He also rarely asked for questions from students. If he did, he never waited for the students to ask the questions. His mode of questioning distracted him from achieving the purpose of using examples to explain the methods of solving the equations. Asking higher order questions and meta-questions as well as the "why?" questions would enable Mwati draw students' attention to exemplary features of the examples and direct students in the procedure for solving exponential equations. In the end the students would generalise from the particular cases.

**Anticipating students' errors and misconceptions**

To identify Mwati's ability to anticipate students' errors and misconceptions during the planning stage, I analysed lesson plans for each lesson he taught. In each lesson plan, Mwati solved all the examples he intended to give to his students. However, he did not solve the lesson problems in various ways. He solved all the equations using correct methods and got correct solutions only, hence, there was no evidence of anticipating errors and misconceptions. As such, he did not indicate any plan of how he would handle such difficulties. In the lesson plan for the second lesson, Mwati indicated that he would correct students' errors but there were no examples of the errors that he thought the students would commit. In the lesson plan for the third lesson, Mwati did not anticipate any errors for the start up and model examples but he anticipated errors for the exercise examples. However, there was no indication of how he would handle the errors probably because he did not display any idea of the types of errors and misconceptions that the students would exhibit. This was also another missed opportunity for Mwati. According to De Garcia (2011), an important aspect of planning a lesson is engaging in solving the lesson problem in various ways. This enables teachers to anticipate students' thinking and the multiple ways they will devise to solve the problem. This also enables a teacher to plan the possible questions he/she may ask to stimulate thinking and deepen students' understanding.

### Identifying students' errors and misconceptions

Mwati identified errors and misconceptions when the students presented results from the exercises. For example, in figure 3, Mwati understood that Maria divided base $5$ by itself. However, it seems he did not understand the reasoning behind the students' error. As such, he erased the students' work and wrote his idea without any explanation about the students' error. This was another missed opportunity for Mwati. Asking Maria to explain her solution process would have helped her and other students with the same difficulty to understand that their reasoning was incorrect and thus to reconstruct a correct conception of the rule of exponents when solving exponential equations.

### Sequencing examples

According to Ball et al. (2008), sequencing examples involves choosing which examples to start with and which examples to use to take students deeper into the content. Both during planning and presentation of the examples, he used start-up examples and model examples as inductive examples and exercises as illustrative examples. The start-up examples in the introduction of each lesson enabled Mwati and students to review the laws of indices. Start-up examples were 'simple examples' as a first step in the development of understanding of a procedure. When he asked questions to help students solve the start-up equation, students answered easily and got correct answers. Model examples followed the start-up examples. Mwati used these examples to take students deeper into the content. The students seemed to have difficulties with the model examples. An interesting model example that Mwati gave in the third lesson is presented in transcript 1.

In each of the three lessons, Mwati gave one model example. This was followed by two or three exercise examples which were done by students either in pairs or individually. The results from the lesson transcript and the lesson plan indicate that he did not go by his plan in the first and second lessons in which he planned to give two model examples but ended up giving one model example in each lesson. Since he was on teaching practice, he had stayed with the students for three days only. Thus, he might not have developed knowledge of the students' ability levels. This finding is consistent with finding by other researchers (Ball et al., 2008) that teacher knowledge develops as the teacher interacts with students.

In the second lesson there was lack of continuity between the first example and the other examples because from the second to the last example, the unknown was the exponent just like in the first lesson. Mwati did not give the students an exercise to practice solving exponential equations in which the unknown was the base. Instead, he gave them equations of which the knowledge and skill were learnt during the first lesson. An exercise of equations of the form $x^a = b$ (where $a$ and $b$ are constants and $b$ can be expressed in base $a$), was necessary in order to help the students generalise the fact that when exponents are the same, the bases are also the same.

**Restructuring tasks**

According to Ball et al. (2008), restructuring tasks involves modifying tasks to make them easier or more difficult. When Mwati related $(2^b)^2 - 9(2^b) + 8 = 0$ to $x^2 - 9x + 8 = 0$, he attempted to restructure the exponential equation to a quadratic form that students were familiar with (see transcript 1). This was the only instance where he attempted to restructure a task. He gave explanations while he solved the two parallel equations but students could not understand. One of the statements by Mwati in transcript 1 indicates that he realised that students did not understand but seemed not to have a remedy to the problem hence he forced them to write an exercise. This might have resulted because Mwati did not anticipate the types of difficulties his students would exhibit. Thus, he was not flexible enough to make the solution process for $(2^b)^2 - 9(2^b) + 8 = 0$ to be transparent for the students.

**Deciding when to pose a new task to further students' learning**

Since Mwati had planned the examples in advance, he could pose a new task after another. However, he posed the new tasks before the students had understood the previous example. Transcript 1 provides evidence of Mwati giving a task to students before they understood the model example. It might be that Mwati was unable to handle students' difficulties that arose during the discussion of the example because he did not anticipate any during lesson planning. As such he forced them to do an exercise before they understood the model example. It could thus be argued that the task Mwati gave did not further learning for some students who did not understand. Listening to students' questions would enable him understand and clear students' difficulties.

The results presented in this paper illuminate the results of a study conducted by Zaslavsky, Harel and Manaster (2006). The researchers found out that in spite of her rather deep mathematical knowledge related to the topic taught and of many manifestations of attentiveness to students and sensitivity to their ways of learning, the preservice teacher also manifested limited ability to unpack the relevant mathematical ideas.

## Conclusion

In this study, Mwati's choice and use of examples was investigated. Findings indicate that Mwati understood to a certain extent how to choose use the examples. Through Mwati's choice and use of the examples, some of his mathematical knowledge came into play. He was very fluent in the way he demonstrated solving exponential equations by equating exponents. However, in spite of his mathematical content knowledge related to the topic, he displayed limited ability to demonstrate solving exponential equations by reducing them to quadratic equations and to attend to students' ways of learning. Mwati also manifested some missed opportunities regarding anticipating, indentifying and handling students' errors. The findings indicate need for improvement in how and why he used the examples in the classroom in order to make the relevant information from the examples facilitate students' learning.

Mwati's choice and use of examples in the three lessons cannot be generalised to all preservice secondary school mathematics teachers at his institution. Further research needs to be carried out to investigate how a larger sample chooses and uses examples for and in the mathematics classroom. However, since Mwati was a particularly bright student and a preservice secondary school teacher who was once a primary school teacher, his presentation of the examples implies that his mathematics teacher education programme does not systematically prepare preservice teachers to deal with the choice and use of examples. It would seem that Mwati had been given the opportunity to develop knowledge that enabled him to do the mathematics himself, but he did not seem to have been given opportunities to develop the knowledge required to carry out tasks that are involved in the work of teaching mathematics. This study proposes to make a step towards teachers' choices and uses of examples. The study proposes that considering their centrality in mathematics teaching and learning, choice and use of examples should be emphasised by the teacher education program in Malawi.

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