Preservice secondary school teachers' knowledge of solving quadratic equations

Florence Thomo Mamba

University of Malawi

fthomo@poly.ac.mw

Abstract

This qualitative study investigated a Malawian preservice secondary school teacher's subject matter knowledge of equations. The study was carried out with one preservice secondary school teacher. Data were generated using a Subject Matter Knowledge paper and pencil test and a task-based interview. The data were analysed using thematic analysis. Results indicate that the preservice secondary school teacher demonstrated some evidence of knowledge of solving quadratic equations, but he was not able to explain why the methods worked. The implications of these findings for mathematics teacher preparation are discussed.

Key words: *Teacher knowledge, preservice, secondary school, quadratic equations*

Introduction

One of the aims of teaching mathematics in Malawi is to improve students, logical reasoning, critical thinking, analysis and abstract thought (Ministry of Education, Science and Technology [MOEST], 2013). Algebra is one of the branches of mathematics that has the ability to promote algebraic thinking which involves problem-solving, representation, and logical reasoning skills and development of abstract thought (Stoelinga & Lynn, 2013). But the Malawi National Examinations Board (MANEB) Chief Examiner's reports indicate that secondary school students fail to develop algebraic thinking at national examinations (MANEB, 2019). Poor teaching practices are highlighted as a major cause of learners' inability to understand geometric proof development (MANEB, 2019). The reports emphasise that due to lack of both content knowledge and pedagogical knowledge, the teachers are not creative in conducting effective lessons to support students' algebraic thinking. Studies conducted in different parts of the world also indicate that despite the importance of algebraic thinking in students' learning, many students face serious chal1lenges in algebraic problem solving and quadratic equations in particular (Makgakga, 2016; Makonye & Nhlanhla, 2014; Ndemo & Ndemo, 2018; Ngoveni & Mofolo-Mbokane, 2019). These studies support MANEB's study by arguing that students' challenges in algebra should be attributed to teachers' classroom inappropriate practices that are also influenced by poor content knowledge (Odumosu & Fisayi, 2018). As a result, the learners memorise the rules without understanding the reasons behind their procedures (Skemp, 1976; Didiş, Baş, & Erbaş, 2011). Use of exploratory teaching strategies is suggested as one way of helping learners to understand relationships between quantities that vary (Star & Rittlejohnson, 2009). This implies that in order to improve students' performance in

Mathematics in general, the teacher should enhance profound understanding and acquisition of algebraic concepts and thinking skills. Thus, improving classroom practices for enhancing students' understanding and development of algebraic thinking skills lies in teacher education and continuous professional development. It is also important to study preservice teachers so that we improve teacher education to prepare teachers with sound mathematical knowledge.

This study builds on the findings from my PhD project (Mamba, 2018) which aimed at investigating preservice teachers' mathematical knowledge for teaching equations. What I found is that the three preservice teachers reported in the study displayed limitations in their problem solving strategies as well as algebraic thinking skills. One of the recommendations made in this PhD study was that studies focusing on the preservice teachers' mathematical knowledge for teaching linear, quadratic and cubic equations respectively would also be beneficial. Therefore this study focused on investigating preservice secondary school Mathematics teachers' knowledge of solving quadratic equations. To achieve this goal, the following research question was addressed: what knowledge of solving quadratic equations do preservice secondary school teachers possess?

Significance of the study

This study shed more light on preservice teachers' knowledge of solving quadratic equations. The study on mathematics teachers' knowledge of quadratic equations is important because it may reveal the necessary approaches that could help pupils have conceptual understanding of algebra in general and quadratic equations in particular. The study could contribute to identifying the most crucial features of subject matter knowledge and support the effective teaching and facilitate substantial growth in pupils" algebraic proficiencies, including the recent emphasis on developing reasons and sense making skills (Graham, Cuoco, and Zimmerman, 2010; Mbewe & Nkhata, 2019).

Teachers' mathematical knowledge of solving quadratic equations

Ball, Thames, & Phelps (2008) contend that teacher knowledge is important for students' learning. Empirical studies have shown that teacher knowledge influences and affects the quality of teaching and learning (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Li, 2011). Studies of beginning and experienced teachers also reveal that teachers' understanding of and agility with the mathematical content affects the quality of their teaching. Many research studies argue that teachers' understanding of the concepts should go beyond what is taught to their learners, because they have to respond to demands different from just being able to solve a problem (Ball, Thames, & Phelps, 2008; Bansilal, Brijlall, & Mkhwanazi, 2014; Ubah & Bansilal, 2018).This implies that teachers must know the content thoroughly in order to be able to present it clearly, to make the ideas accessible to a wide variety of students, and to engage students in challenging work (Ball, Thames, & Phelps, 2008). Besides, teachers need to know and be able to do more than doing the mathematics for themselves so that they are able to support students' learning of algebra and equation solving. Shulman argues;

for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation (Shulman, 1986: 9).

This description emphasises on the importance of strong mathematical knowledge of 'the most regularly taught topics in one's subject area', because finding ways of explaining the concept so that it is comprehensible to others requires that one must understand the concept itself very well. Ball et al. (2008: 399) refer to 'common content knowledge' as the knowledge of 'a kind used in a wide variety of settings – knowledge not unique to teaching'. This implies that common content knowledge is the knowledge that teachers themselves mediate with the learners in the class. In order to carry out the task of teaching, a teacher needs a more profound understanding of the concept to be taught. A teacher should know the various definitions of concepts, be able to select relevant examples and exercises, be able to choose the sequence in treating a specific topic, be able to recognise the usefulness of particular representations over others in certain circumstances, and distinguish between correct and unproductive strategies used by learners when solving problems. All these tasks of a teacher depend on a potent common content knowledge that they are mediating with their learners Bansilal, Brijlall & Nkhwanazi, 2014). Some of these demands are associated with pedagogical content knowledge (PCK) (Shulman, 1986).

Wu (2005: 9) argues that often 'a well-intentioned pedagogical decision in the classroom can be betrayed by faulty content knowledge'; thus, emphasising the importance of content knowledge in the teaching of Mathematics. Li (2011) studied a teacher's mathematical knowledge for teaching algebraic routines and focused on quadratic equations. Results indicated that the teacher's knowledge of the equation solving routines played the most crucial role in shaping the lessons and potentially promoting mathematical proficiency in a balanced approach. Bansilal (2012b) investigated a teacher's poor mathematics content knowledge and found that the teacher's explanations were often incoherent and illogical. In addition, Bansilal et al. (2014) explored high school practising teachers' common content knowledge. The researchers found that the practicing teachers were struggling with the mathematics content that they were teaching. The results showed that the teachers did not know sufficient school mathematics and consequently, this led to poor teaching and learning of Mathematics in their classrooms. Mbewe and Nkhata (2019) also studied secondary school teachers' Mathematics Knowledge for Teaching quadratic equations. The researchers found that the teachers mathematical knowledge for teaching ability to generate different solutions, to address pupils‟ difficulties and misconceptions and to choose appropriate examples to teach quadratic equations were based on the depth of the subject matter which played a critical role in their reasoning and decision making in specific contexts.

While most studies on teacher knowledge focused on teachers Mathematical Knowledge for Teaching (MKT) in general (Li, 2011; Bansilal, 2012b; Ubah & Bansilal, 2018; Mbewe & Nkhata, 2019), studies on teacher content knowledge are scanty, particularly those focusing on quadratic equations. In addition, research reports on mathematical knowledge for teaching among Malawian preservice secondary school mathematics teachers are less available. The purpose of this study was thus to investigate a preservice secondary school teacher's knowledge of solving quadratic equations. Focusing on preservice teachers will inform teacher education so as to produce teachers who are equipped with knowledge and skills to teach Mathematics effectively. The results from this study could thus inform preservice teacher educators about the content of Mathematics teacher preparation.

Quadratic equations

The understanding of quadratic equations is critical for students' understanding of further mathematics such as polynomials and calculus (Kotsopoulos, 2007; Didis & Erbas, 2011). Quadratic equations play an important role in the development of mathematical concepts in that it cuts across a range of mathematics content domains including those of Algebra and Geometry (National Council of Teachers of Mathematics [NCTM], 2000). Although quadratic equations take an important role in secondary school algebra curricula around the world, research indicates that quadratic equations are one of the conceptually challenging aspects of the high school curriculum (Kotsopoulos, 2007). A number of researchers agree that the concept of quadratic equations poses challenges to students. (Makonye & Nhlanhla, 2014; Makonye & Matuku, 2016; Makgakga, 2016; O'connor & Norto, 2016). Further, it appears that studies concerning teaching and learning quadratic equations are quite scarce in algebra education research (Vaiyavutjamai & Clements, 2006; Kieran, 2007; Dids & Erbas, 2011). In a review of literature, Ubah and Bansilal (2018) found a small amount of research on pre-service teachers' understanding of the graphs of quadratic functions and links between the graphical and symbolic representations. The studies indicated that learners at all levels of education have difficulties in determining quadratic relationships. As such, there was a need to explore the of pre-service mathematics teachers' understanding of quadratic equations.

Theoretical considerations

Two constructs guide the theoretical framework for this study: Shulman's conception of teacher knowledge and equations and Ball et al.'s (2008). Shulman (1986) developed a framework for transforming subject matter into pedagogy. He divided teacher knowledge into three categories: subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curricular knolwedge (CK). Shulman argues that in teacher development, educators and researchers should focus on both SMK and PCK in order to help preservice teachers to transform from expert students to novice teachers. On SMK, Shulman argues that a teacher should not only understand that something is so, but also why it is so. Knowing 'that' involves knowledge of rules, algorithms, procedures, concepts and principles that are related to specific mathematical topics in the school curriculum. Knowing 'why' includes knowledge which pertains to the underlying meaning and understanding of why things are the way they are (Even & Tirosh, 1995). Knowing 'why' also affects teacher's decisions about the presentation of the subject matter. On PCK, "teachers need to understand the various representations of functions; to move flexibly between different representations; to know different methods of solving problems; to know which method is easier than the other; to recognise whether different methods are equivalent or not; and, to be able to explain under what conditions certain procedures are appropriate or not" (Ubah & Bansilal, 2018, p. 848).

In addition to Shulman's teacher knowledge framework, Ball, thames and Phelps (2008) developed the Mathematical Knowledge for Teaching (MKT) framework which elaborates the use of the Shulman's (1986) subject matter knowledge, pedagogical content knowledge and curriculum knowledge categories, organising and defining them in a different way. There are six elements of MKT: Common Content Knowledge CCK), Specialised Content Knowledge (SCK), Horizon Content Knowledge (HCK), Knowledge of Content and Students (KCS), Knowledge of Content Teaching (KCT) and Knowledge of Content and the Curriculum (KCC). On CCK is the mathematical knowledge teachers want their students to develop (Hill, Sleep, Lewis and Ball, 2007). This knowledge of subject matter comprises interrelated concepts, processes, properties, justifications, applications, comparisons, methods, and representations and sequences of reasoning, both as an academic discipline and as a course of study (Li, 2011).

The second construct of the theoretical framework for this research, the study of equations, largely draws from the work of Kriegler (2007). Kriegler developed a framework for algebraic thinking which is based on many years of work. Kriegler asserts that algebraic thinking is organised into two major components: the development of mathematical thinking tools and the study of fundamental algebraic ideas. Of these two components, mathematical thinking tools informed this study. Mathematical thinking tools are analytical habits of mind. They are organised around three topics: problem solving skills, representation skills, and quantitative reasoning skills. These thinking tools are essential in many subject areas, including mathematics; and quantitatively literate citizens utilise them on a regular basis in the workplace and as part of daily living (Kriegler, 2007). Thus, to develop and enhance students' algebraic thinking, preservice secondary school teachers themselves need to exhibit high levels of algebraic thinking skills and be able to articulate what it is that they are doing.

METHODOLOGY

This was a qualitative descriptive case study involving three preservice secondary school mathematics teachers named Mwati, Dinga and Mwatha (pseudonyms). They were diploma in education student in a three-year programme at a college of education in Malawi. Mwati and Mwatha were trained as primary school teachers and had taught in primary schools for two years before joining the secondary school teacher education programme while Dinga had not received any teacher training and had not taught before joining the secondary school teacher training programme. When data for this study were generated, they were in the final year of study. Being the best students in their class, they were considered “information-rich” cases for in-depth study (Yin, 2014).

A subject Matter knowledge test and a video-recorded semi-structured interview were used to generate the data. In the Subject Matter Knowledge test, participants were required to solve two quadratic equations. During the interview, they were given one quadratic equation to solve. The video recorded interview allowed for multiple, in-depth rounds of analysis of the data (Girden & Kabacoff, 2011). Test and interview tasks reported here were adapted from a Malawi secondary school mathematics textbook (Gunsaru & Macrae, 2001) and were piloted before the main study. The first equation was $5x^2 + 8x - 2 = 0$. The participants were required to solve it using any method of their choice. For the quadratic equation $3x^2 + 7x - 6 = 0$, the participants were asked to solve and illustrate the methods which they knew. They were also asked to explain the method they thought was better and why. Thus the participants’ use of problem solving strategies and their ability to explore multiple approaches to a problem were assessed using this item. During the interview, the participants were asked to solve the equation $x^2 = 2x + 8$ using three methods of their choice one after another. This quadratic equation required the participants to change it to standard form before beginning to solve it. A quadratic equation is in standard form if it is in the form $ax^2 + bx + c = 0$.

Data were analysed using thematic analysis (Ritche & Lewis, 2003). Themes were developed *a priori* from the theoretical framework as well as *a posteriori* the data. During the initial coding, some themes that were not in the theoretical framework were emerging from the data. Some of these themes were incorporated into the theoretical framework while others were regarded as separate categories of the characteristics of preservice teacher‘s knowledge of solving quadratic equations (Boyatzis, 1998).

Results and discussion

Results from the study show that the preservice teachers displayed different kinds of knowledge of solving the equations. For instance, they displayed knowledge of multiple approaches to the quadratic equation, knowledge of various representations of quadratic equations by moving flexibly between different representations, knowledge of relationships, knowledge of different methods of solving quadratic equations and knowledge of the procedure of solving quadratic equations.

Knowledge of multiple approaches to quadratic equations

The findings show that the preservice teachers were able to explore multiple approaches to solving quadratic equations. Among the methods of solving the quadratic equations, each preservice teacher explained which method was the easiest for him to use.

The case of Mwati

To solve the quadratic equation $5x^2 + 8x - 2 = 0$, Mwati first factorised the quadratic and saw that factor method did not work. Then he realised that there are no factors of -10 whose sum is $+8$, the coefficient of x. As a result, he used the quadratic formula but did not explain the reasoning behind his ideas despite being asked to explain.

In another quadratic equation $3x^2 + 7x - 6 = 0$. Mwati used factorisation method and listed the other methods. In the end, he chose the method he thought was better. In using this method, he explained that it was easy for him to find the factors of -18 that added to $+7$. Although Mwati explained why he chose factorisation method as a better method to solve this quadratic equation, he did not explain why he did not choose the others. For this reason, I followed up this point during the interview and asked him why he observed that factorisation is a better method than the others. Mwati noted that he would use factorisation because it saves time. For example, he explained that using a graph would be difficult and requires coming up with a table. He added that even coming up with the values of x is difficult because the values have to be chosen arbitrarily. Thus the students can choose maybe, 100 or the range of 100 to 110 or any other numbers and this may make the problem harder. Commenting on completing the square, Mwati observed that sometimes students may not understand why are we add the square of half of the coefficient of x on both sides of the equation.

5. Solve the quadratic equation $3x^2 + 7x - 6 = 0$. Illustrate all the methods which you know. Which method do you think is better and **why?**

Soln

Solving $3x^2 + 7x - 6 = 0$

$3x^2 + 9x - 2x - 6 = 0$

$3x(x+3) - 2(x+3) = 0$

$(x+3)(3x-2) = 0$

either $x + 3 = 0$

$x = -3$

or $3x - 2 = 0$

$x = 2/3$

The methods that can be used to solve the above problem are:

(i) Factorization method

(ii) Graphical method

(iii) Completing the Square

(iv) Use a formula that States $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

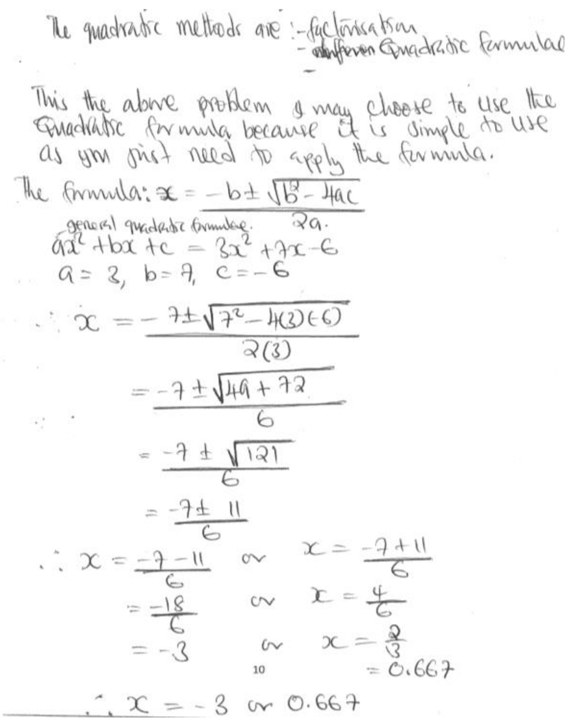
The better method for Solving above problem is factorisation because the factors of -18 where added are easily providing us with the coeffient of x that is $(+7)$.

Figure 1: Mwati's solution process to the equation $3x^2 + 7x - 6 = 0$

Teacher Knowledge	Examples
Knowledge of multiple approaches to a problem ability to explore the approaches	**Mwati**: Yes, because this is factorisation, I will also use quadratic formula to solve the equation **Mwati**: I think the problem can also be solved by quadratic formula … We can also use completing the square.

The case of Dinga

When solving the quadratic equation $5x^2+8x-2=0$, Dinga used factorisation method incorrectly. After getting –10 from $5x$ – 2, Dinga had $5x^2+5x-2x-2=0$ but he did not realise that there are no factors of –10 that can add or subtract to give +8, the coefficient of x. This made him come up with $x=1$ or $x=\frac{2}{5}$. This task could be solved either by completing the square, or by the quadratic formula. Dinga solved the equation $3x^2+7x-6=0$ by using the quadratic formula only. Nonetheless, he was required to solve the equation using all appropriate methods.


The quadratic methods are: - factorisation
- Quadratic formulae

This the above problem I may choose to use the Quadratic formula because it is simple to use as you just need to apply the formula.

The formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

general quadratic formulae.

$ax^2+bx+c=3x^2+7x-6$

$a=3,\ b=7,\ c=-6$

$\therefore x=\frac{-7\pm\sqrt{7^2-4(3)(-6)}}{2(3)}$

$=\frac{-7\pm\sqrt{49+72}}{6}$

$=\frac{-7\pm\sqrt{121}}{6}$

$=\frac{-7\pm 11}{6}$

$\therefore x=\frac{-7-11}{6}$ or $x=\frac{-7+11}{6}$

$=\frac{-18}{6}$ or $x=\frac{4}{6}$

$=-3$ or $x=\frac{2}{3}$

10

$=0.667$

$\therefore x=-3$ or 0.667

Figure 2: Dinga’s solution process to the equation $3x^2+7x-6=0$

Teacher Knowledge	Examples
Knowledge of multiple approaches to a problem ability to explore the approaches	**Interviewer**: Thank you. Would you solve the same problem using another method? **Dinga**: Let me see (he scribbled on paper to see if he would find a method to use) (He solved the equation using and the quadratic formula but stated that he forgot the other method meaning completing the square) He did not mention graphical method

The case of Mwatha

Mwatha also solved the equation $5x^2 + 8x - 2 = 0$ using quadratic formula. In his solution process, Mwatha explained that the quadratic equation he was solving was of the form $ax^2 + bx + c = 0$ $ax^2 + bx + c = 0$. He further said that quadratic expressions and equations can be solved using three methods. He said that the first is by factorisation. Where factorisation does not work, one can use completing the square method or the quadratic formula.

Mwatha also solved the quadratic equation $3x^2 + 7x - 6 = 0$ by factoring, completing the square and the quadratic formula. He partially explained the procedure in factorisation and completing the square methods. Mwatha explained that the best method among the listed and used is quadratic formula. By explaining that quadratic equations can also be solved using quadratic equations, Mwatha meant quadratic formula. He explaned that in the quadratic formula, one can just substitute the values in formula without problems. During the interview, Mwatha explained that factorisation method is the best method because the other methods are long and time consuming. He added that most students would avoid completing squares and that others find it difficult to memorise the quadratic formula.

5. Solve the quadratic equation $3x^2 + 7x - 6 = 0$. Illustrate all the methods which you know. Which method do you think is better and **why?**

(c) Solving $3x^2+7x-6=0$ using factorisation method:

Soln

$3x^2+7x-6=0$ -- multiply coefficient of x^2 and -6 $(3\times 6)=18$

$3x^2+9x-2x-6=0$ (find factors of 18 which when multiplied will give 18 and when added will give 7.

$3x(x+3)-2(x+3)=0$ --- factorising

$(3x-2)(x+3)=0$

$\therefore (3x-2)=0$ or $x+3=0$

$\therefore x=2/3$ or $x=-3$.

(b) Solving $3x^2+7x-6=0$ by completing squares.

$3(x^2+7/3x-2)=0$ ...making coefficient of $x^2=1$.

$x^2+7/3x+(7/6)^2-(7/6)^2-2=0$

$x^2+(7/6)^2=2+\frac{49}{36}$

$(x+7/6)^2=\frac{121}{36}$

$x=-7/6\pm\sqrt{\frac{121}{36}}$

$x=-7/6\pm\frac{11}{6}=\frac{-7\pm 11}{6}$

10

$x=4/6=2/3$ or $x=-3$

$\therefore x=2/3$ or $x=-3$.

c. the quadratic equation $3x^2+7x-6=0$ can also be solved using quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$a=3,\ b=7,\ c=-6$ -- Substituting we have;

$$x=\frac{-7\pm\sqrt{(7)^2-4(3)(-6)}}{2(3)}$$

$$x=\frac{-7\pm\sqrt{49+72}}{6}$$

$$x=\frac{-7\pm\sqrt{121}}{6}$$

$$x=\frac{-7\pm 11}{6}$$

$x=2/3$ or $x=-3$.

Figure 3: Mwatha's solution process to the equation $3x^2 + 7x - 6 = 0$

Teacher Knowledge	Examples
Knowledge of multiple approaches to a problem ability to explore the approaches	**Mwatha**: Because we are trying to find the values of x. So we should try to find the factors and these factors should be equated to zero. **Mwatha**: The second method that comes in my mind is using the quadratic formula. **Mwatha**: Completing squares. So I will solve using completing squares. So we have $x^2 - 2x - 8 = 0$

Knowledge of multiple approaches to a problem and exploring the approaches gives teachers and students opportunities to develop good problem-solving skills and experience the utility of mathematics (Kriegler, 2007). When solving the quadratic equations the preservice teachers used rules of algebra to solve for the unknowns and thus indicating deductive reasoning skills.

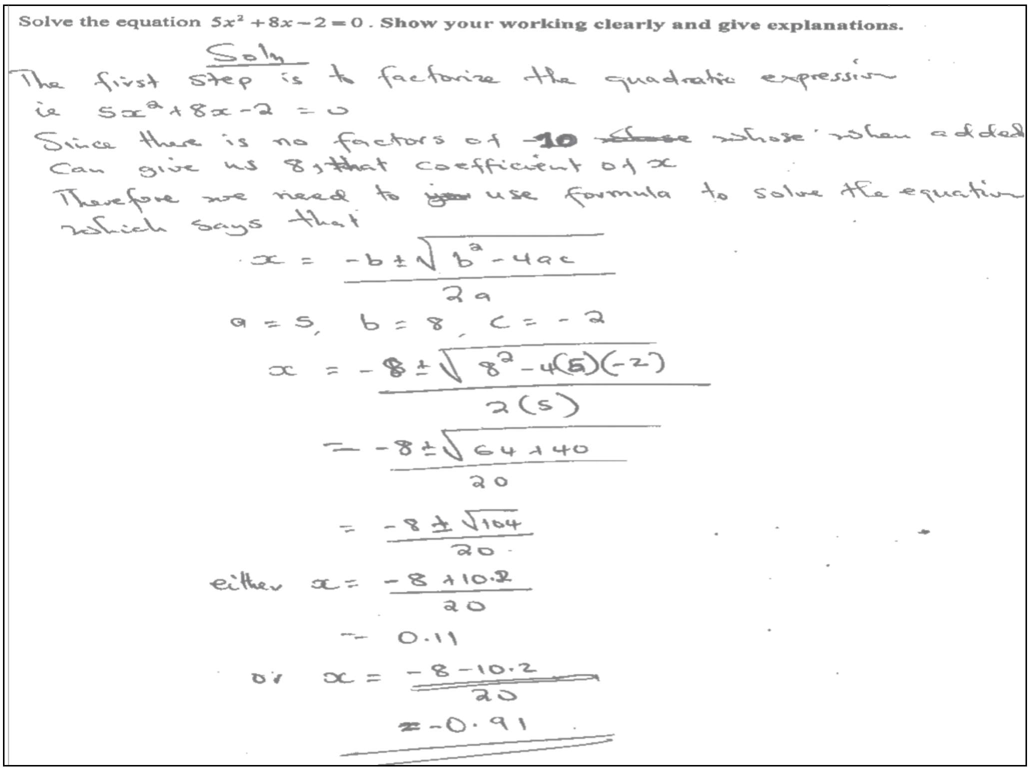
While Mwati and Mwatha noticed that the quadratic equation $5x^2 + 8x - 2 = 0$ could not be solved by factorisation method, Dinga used factorisation method incorrectly because this method was not appropriate for this task. This task could be solved either by completing the square or by the quadratic formula. It was also possible to solve it graphically. It seems that Dinga did not realise that there are no factors of $-10x^2$ that could give $+8x$. Mwati and Mwatha explained the limitation of factorisation method to solve this equation. Failure to recognise the limitation of factorisation method in this quadratic equation indicates Dinga's limited problem solving skills and CCK.

For the equation $3x^2 + 7x - 6 = 0$, Mwati solved using factorisation method and Dinga, solved using the quadratic formula. Both Mwati and Dinga listed the other methods that could be used to solve this equation. Unlike his colleagues, Mwatha solved the same equation using three methods; factor method, completing the square and by the quadratic formula. Considering their choices of a better method among the ones they used and mentioned, Mwati explained that a better method is factorisation while Dinga and Mwatha chose the quadratic formula. Dinga and Mwatha explained that that substituting in the formula is easy. Mwati's choice of factorisation as a simple and better method was consistent with his choice in the interview. Dinga's and Mwatha's test and interview results were inconsistent. During the interview, both Dinga and Mwatha explained that factorisation was an easier method to use Mwati's and Mwatha's ability to explore multiple approaches was also evident by justifying why their procedures failed. When solving an equation $5x^2 + 8x - 2 = 0$, Mwati first of all attempted to use factorisation method. After he realised that factorisation method failed, he used the quadratic formula. Mwatha also explained that the equation could best be solved by quadratic formula but he did not justify his choice of the method. Being able to explore multiple approaches to a problem is a problem solving skill (Kriegler, 2007). However, the preservice teachers displayed procedural knowledge. Details on the preservice teachers' procedural knowledge are presented in section 5.1.3.

The preservice teachers also displayed limitations in their problem solving strategies. They attempted to use problem solving strategies to solve the river current problem, the endangered species problem and the equation $x^2 = 2x + 8$.

Knowledge of which method of solving quadratic equations is the easiest

Findings indicate that Mwati was able to solve quadratic equations during the test and interview. He used factor method by difference of two squares, factorisation by trial and error, completing the square, the quadratic formula and by graph. During the test, Mwati solved the equations $25m^2 - 1 = 0$, $5x^2 + 8x - 2 = 0$ and $3x^2 + 7x - 6 = 0$. For instance, to solve the equation $3x^2 + 7x - 6 = 0$ in Figure 1, Mwati firstly factorised the quadratic expression on the left side and saw that factor method did not work. Then he realised that there are no factors of -10 whose sum is $+8$, the coefficient of x and thus, used the quadratic formula. During the interview he solved the equation $x^2 = 2x + 8$ by factor method, quadratic formula, completing squares and by graph in that order. Table 1 illustrates Mwati's exploration of the methods of solving the quadratic equation $x^2 = 2x + 8$.



Solve the equation $5x^2 + 8x - 2 = 0$. Show your working clearly and give explanations.

Soln

The first step is to factorize the quadratic expression

ie $5x^2 + 8x - 2 = 0$

Since there is no factors of -10 whose when added Can give us 8, that coefficient of x

Therefore we need to use formula to solve the equation which says that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 5, \quad b = 8, \quad c = -2$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(5)(-2)}}{2(5)}$$

$$= \frac{-8 \pm \sqrt{64 + 40}}{20}$$

$$= \frac{-8 \pm \sqrt{104}}{20}$$

either $x = \frac{-8 + 10.2}{20}$

$= 0.11$

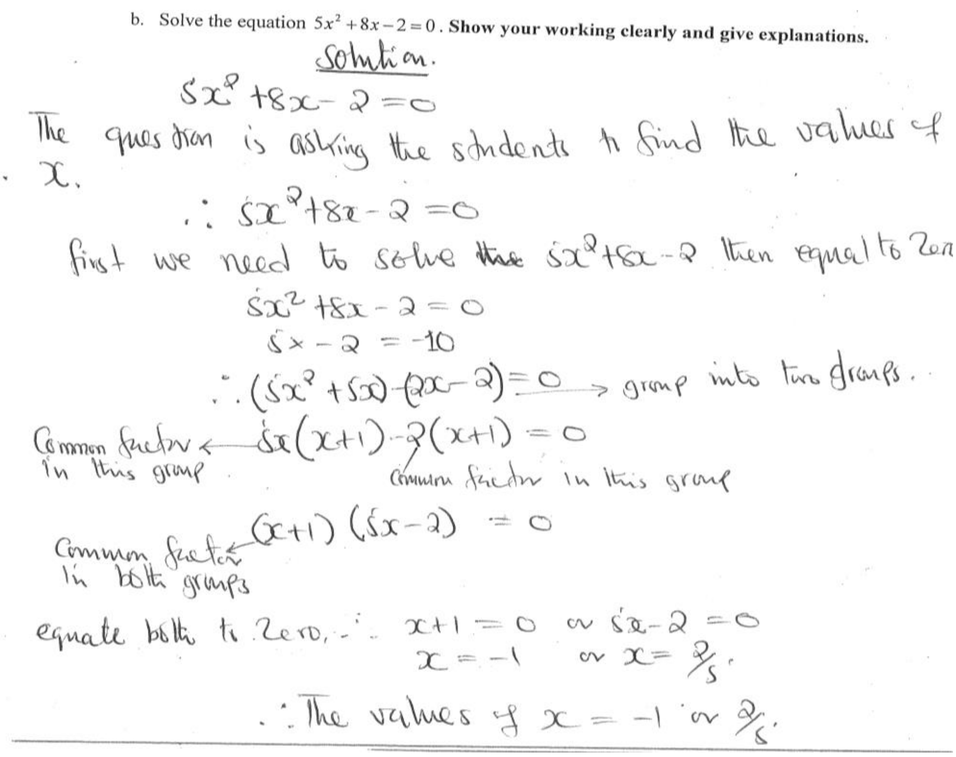
or $x = \frac{-8 - 10.2}{20}$

$x = -0.91$

Figure 1: Mwati's solution process to the quadratic equation $\mathbf{3x^2 + 7x - 6 = 0}$

Although Mwati explained which method was the easiest both during the test and interview; we cannot assume that he has well-articulated subject matter knowledge. He did not indicate any knowledge of why he carried out particular actions in his solution processes. Thus Mwati lacks an important aspect of subject matter knowledge. Being able to use an algorithm to solve a problem and failing to justify steps in the algorithm is an indication of procedural knowledge (Hiebert &

Dinga


b. Solve the equation $5x^2 + 8x - 2 = 0$. **Show your working clearly and give explanations.**

Solution.

$5x^2 + 8x - 2 = 0$

The question is asking the students to find the values of x.

$\therefore 5x^2 + 8x - 2 = 0$

first we need to solve ~~the~~ $5x^2 + 8x - 2$ then equal to zero

$5x^2 + 8x - 2 = 0$

$5x - 2 = -10$

$\therefore (5x^2 + 5x) - (2x - 2) = 0$ → group into two groups.

Common factor in this group ← $5x(x+1) - 2(x+1) = 0$

Common factor in this group

Common factor in both groups ← $(x+1)(5x-2) = 0$

equate both to zero, $\therefore$ $x + 1 = 0$ or $5x - 2 = 0$

$x = -1$ or $x = \frac{2}{5}$.

$\therefore$ The values of $x = -1$ or $\frac{2}{5}$.

Mwatha

After he solved the quadratic equation $3x^2 + 7x - 6 = 0$ by factorisation, completing the square and quadratic formula, Mwatha explained that using the quadratic formula was the easiest because substituting numbers into the formula is easy.

Quadratic equations can be solved using the above methods that is factorization, completing the square and using the quadratic equation. I think the quadratic equation is the best methods to use since you can easily substitute the values directly in the formula and work out the solutions unlike the other methods. They are likely to mislead the problem solver because they are a bit complex. One can easily add uncessary information or miss out information.

Lefevre, 1986). Even and Tirosh (1995) also argue that "knowing that" though certainly important is not enough. "Knowledge which pertains to the underlying meaning and understanding of why things are the way they are, enables better pedagogical decisions" (Even and Tirosh, 995; pp. 9).

Discussion

The findings reveal that the preservice teachers were able to explore multiple approaches to quadratic equations, chose methods of solving quadratic equations they found easy and justified their methods of choice.

By explaining that that he used factorisation to solve the equation $3x^2 + 7x - 6 = 0$ because he easily found factors of $-18x$ which gave him the middle term in the equation, Mwati justified his response. Similarly, by explaining that the easiest method to use to solve the quadratic equation was quadratic formula because one just needs to apply the formula, Dinga also justified his response. Similarly, in Figure -----, Mwatha explained why he felt that the quadratic formula was the easiest method of all.

Although the preservice teachers displayed some conceptual understanding during the SMK test and the interviews, we cannot assume that they had well-articulated SMK. There were some things that they needed to improve. For instance, they mostly explained the 'what' of the solution process but not the 'why'. In their attempts to solve the equations where necessary, the preservice teachers relied much on explaining the procedures they used. They did not give any conceptual justifications of the procedures. This problem was consistent in the test items and interviews. They indicated little knowledge of why they carried out particular actions in their solution processes. When they attempted to give reasons, the reasons had some limitations or were incorrect.

Although Mwati was able to solve the given equations on the SMK test and interviews, he did not appear to be able to explain why the methods worked. When Dinga explained the procedures for solving the equation $x^2 = 2x + 8$ by factor method and quadratic formula, he was trying to give mathematical explanations. Similarly, the results presented in section 4.1.3 indicate Mwatha's attempt to explain a procedure. In his solution processes, Mwatha explained the 'what' of the procedure not the 'why'. From the beginning to the end of the solution processes, he did not give the reasons behind moving from one step to another. As

Shulman argues, “a teacher should not only understand that something is so, but also why it is so”. Even and Tirosh (1995) further observe that the most basic level of subject matter is *knowing that* and *knowing why*. Knowing *that* involves knowledge of rules, algorithms, procedures, concepts and principles that are related to specific mathematical topics in the school curriculum. Knowing *why* includes “knowledge which pertains to the underlying meaning and understanding of why things are the way they are” (p. 9). Knowing why also affects teacher’s decisions about the presentation of the subject matter. Thus the preservice teachers lack an important aspect of SMK. Being able to use an algorithm to solve a problem and failing to justify steps in the algorithm is an indication of procedural knowledge (Hiebert & Lefevre, 1986). It could thus be expected that the preservice teachers lack the knowledge necessary for teaching equations.

The preservice teachers might not have justified their procedures and ideas probably because reasoning and justifying are advanced mathematical thinking processes (Tall, 2002). Thus the preservice teachers might not have been at the level of advanced mathematical thinking. It might also not be a normal practice to justify among Malawian preservice teachers.

CONCLUSION

In this study, Mwati’s subject matter knowledge of quadratic equations was investigated through a paper and pencil test and a task-based interview. Findings show that Mwati displayed success when solving the equations using factorisation, quadratic formula, completing the square and by graph. When solving the equations, he was able to explore multiple approaches to a problem, to use rules of algebra to solve the equations and hence displayed deductive reasoning. However, Mwati displayed procedural knowledge such that he was able to solve the given equations, but did not appear to be able to explain why the methods worked. It could thus be expected that Mwati lacks the knowledge necessary for teaching equations effectively to his students. Since this apparent lack of knowledge could be observed with the best students, it appears that Malawian teacher education needs to emphasise on the ’why’ questions more and go beyond emphasising on the procedural aspects of equation solving. This aspect is lacking in contemporary education of secondary school mathematics teachers in Malawi. With emphasis on ’why’ questions, teachers will be able to elicit and interpret students’ thinking, identify misconceptions, provide tasks and pose questions that will guide students’ interpretations of mathematics. The teachers will also be able to find appropriate strategies for inducing cognitive conflict that will enable students to deconstruct their ‘naïve theories‘ and reconstruct correct mathematical conceptions.

References

Ball, D. L., Thames, M. H. & Phelps, G. C. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.

Bansilal, S., Mkhwanazi, T., & Brijlall, D. (2014). An exploration of the common content knowledge of high school mathematics teachers. *Perspectives in Education, 32*(1), 34-50.

Boyatzis, R. E. (1998). *Transforming qualitative information: Thematic analysis and code development.* Thousand Oaks, CA: Sage.

Didiş, M. G., Baş, S., & Erbaş, A. (2011). Students' reasoning in quadratic equations with one unknown. In *The Seventh Congress of the European Society for Research in Mathematics Education (CERME-7)* (pp. 479-489).

Even, R. & Tirosh, D. (1995). Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject matter. *Educational Studies in Mathematics, 29(1)*, 1-20.

Girden, E. R. & Kabacoff, R. I. (2011). *Evaluating research articles*. New Delhi: SAGE.

Gunsaru, C. M. & Macrae, M. F. (2001). *New Secondary Mathematics. Form 2* (2^{nd} Ed.). Blantyre: Longman Malawi.

Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M. Phelps, G. C. Sleep, L. & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An Exploratory Study. *Cognition and Instruction*, 26, 430–511.

Kriegler, S. (2007). Just what is algebraic thinking? Introduction to algebra: *Teacher handbook*, 7-18. *pp. 1-16.*

Kotsopoulos, D. (2007). Unravelling student challenges with quadratics: A cognitive approach. *Australian Mathematics Teacher, The*, *63*(2), 19-24.

Li, X. (2011). Mathematical knowledge for teaching algebraic routines: A case study of solving quadratic equations. *Journal of Mathematics Education*, *4*(2), 1-16.

Makgakga, S. (2016). Errors and misconceptions in solving quadratic equations by completing a square. *Mathematics Education.* diakses, 8.

Makonye, J.P., & Matuku, O. (2016). Exploring Learner Errors in Solving Quadratic Equations. *International journal of educational sciences*, *12*(1), 7-15.

Makonye, J. & Nhlanhla, S. (2014). Exploring 'Non-Science' Grade 11 Learners' Errors in Solving Quadratic Equations. Mediterranean Journal of Social Sciences, Vol 5 (27), pp. 634-644.

Malawi National Examinations Board (2019). *Mathematics Chief Examiners' Reports*. Zomba (Unpublished).

Ndemo, O. & Ndemo, Z. (2018). Secondary school students' errors and misconceptions in learning algebra. *Journal of Education and Learning (EduLearn)*, *12*(4), 690-701.

Ngoveni, M. A., & Mofolo-Mbokane, B. (2019). Students misconceptions in algebra: A case of National Certificate (vocational) level 2 Engineering Mathematics students. *Association for Mathematics Education of South Africa*, *1*, 232.

O'Connor, B. R., & Norton, S. (2016). Investigating Students' Mathematical Difficulties with Quadratic Equations. *Mathematics Education Research Group of Australasia*.

Odumosu, O., & Fisayi, A. (2018). Teachers' content and pedagogical knowledge on students' achievement in algebra. *International Journal of Education and Research*, *6*(3), 83-94.

Patton, M.Q. (2002). *Qualitative research and evaluation methods* (3^{rd} edition). Thousand Oaks, CA: SAGE Publications.

Ritchie, J. & Lewis, J. (2003). *Qualitative research practice: A guide for social science students and researchers*, London: SAGE.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15 (2)*, 4-14.

Star, J. R., & Rittle-Johnson, B. (2009). Making algebra work: Instructional strategies that deepen student understanding, within and between representations. *ERS Spectrum*.

Stoelinga, T. & Lynn, J. (2013). Algebra and the underprepared learner. *Policy Brief*, *2*(3).

Ubah, I. J. A., & Bansilal, S. (2018). Pre-service mathematics teachers' knowledge of mathematics for teaching: quadratic functions. *Problems of Education in the 21st Century*, *76*(6), 847.

Wu H 2005. Must content dictate pedagogy in mathematics education? Talk given at California State University, USA.

Yin, R. K. (2014). *Case Study: Research, Design and Methods*. (5th Ed.). Los Angales: Sage Publications.