A PRESERVICE SECONDARY SCHOOL TEACHER'S MATHEMATICS FOR TEACHING EXPONENTIAL EQUATIONS

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This qualitative study investigated a Malawian preservice secondary school teacher's mathematics for teaching exponential equations. The study was carried out with one preservice teacher. Data was generated using video lesson recordings and analysed using thematic analysis. The findings reveal that the preservice teacher was not able to unpack or decompress equation solving to make it accessible to students. Implications of these findings for mathematics teacher preparation are discussed.

INTRODUCTION

Teacher knowledge is important for students' learning. Empirical studies have shown that teacher knowledge influences and affects the quality of teaching and learning (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). Studies of beginning and experienced teachers also reveal that teachers' understanding of and agility with the mathematical content affects the quality of their teaching. Thus, teachers must know the content thoroughly in order to be able to present it clearly, to make the ideas accessible to a wide variety of students, and to engage students in challenging work (Ball, Thames & Phelps, 2008). Several researchers argue that knowing mathematics for teaching requires knowing in detail the topics and ideas that are fundamental to the school curriculum and beyond (Ibid). In order to support students' learning of algebra and equation solving, teachers need to know and be able to do more than doing the mathematics for themselves. Despite this call by researchers, research reports on mathematical knowledge for teaching among Malawian preservice secondary school mathematics teachers are less available. Hence, this study examined the following question: What mathematics for teaching is displayed by a Malawian preservice secondary school mathematics teacher? The results from this study could inform preservice teacher educators about the content of mathematics teacher preparation.

THEORETICAL BACKGROUND

Malawi is one of the countries which are experiencing challenges in the education system. Secondary school students' performance in national examinations continues to be poor with only around 50 percent of students passing end-of-cycle examinations (Ministry of Education, 2008). Analyses of national examinations chief examiners' reports for years 2008 to 2013 also indicate students' poor performance in algebra in general and equations in particular. While explanations are available concerning system failure, (Ministry of Education Science and Technology, 2008), teacher knowledge is also vital in the development of students' conceptual understanding. Thus, in order to improve students' performance in mathematics in general, teachers need to possess adequate and appropriate mathematics for teaching.

In teacher education, teaching practice is the most important aspect through which preservice teachers' performance is produced. As such, Malawian preservice secondary school teachers practise teaching partly in the course of their final semester and finally, for twelve weeks after their theory courses. In this paper, I describe a preservice secondary school teacher's mathematics for teaching that was revealed during teaching practice.

The theoretical framework for this study is that mathematical knowledge for teaching is situated in the practice of teaching (Ball, et. al.: 2008). In studying teaching, the study draws on Ball et al. (2008) who suggest different tasks teachers need to perform and go about their work in the mathematics classroom. Kazima, Pillay and Adler (2008) condensed these into six categories. The six categories of tasks for teaching are as follows:

1. Defining: attempts to provide a definition
2. Explanations: teachers explain an idea or procedure
3. Questioning: teacher asks questions to move the lesson on
4. Representations: teachers represent ideas and in various ways
5. Working with learners' ideas: teachers engage with both expected and unexpected learners' mathematical ideas
6. Restructuring tasks: teachers change set tasks by scaling them either up or down

In this study, one preservice teacher's ability to define, explain, question, work with students' ideas and restructure tasks was investigated. I assumed a Vygotskian conception of mathematics teaching and learning as socially mediated (Vygotsky, 1978). Vygotsky believes that intellectual development is essentially a process of making meaning with others. He argues for the central role of discourse at all levels of education. Thus for teaching and learning to be meaningful for students, there should be collaboration between the teacher and the students and among students themselves.

METHOD

According to Marshall and Rossman (1999), this research falls into the qualitative design with case study approach. Merriam (2009) asserts that qualitative research focuses on process, meaning and understanding. In order to arrive at judgments regarding preservice teachers' MKT, *"richly descriptive data"* were needed (Merriam, 2009, p. 14). Words, therefore, were used instead of numbers to convey the outcomes of the research findings.

One male preservice secondary school mathematics teacher, Mwati (pseudonym), participated in the study. He was a Diploma in education student at a college of Education. The course program takes three years. Every year the college produces about 300 Diploma and Bachelors' Degree secondary school teachers through face to face programme and 200 diploma teachers through distance education programme. These teachers are posted to various secondary schools in the country. Mwati was enrolled for the face to face programme. He was aged between 30 and 40. He was originally trained as a soldier. Later, he was trained as a primary school teacher for two years under the Integrated Primary Teacher Education program (IPTE). The IPTE program in Malawi is run in such a way that students learn theory for one year and they go for teaching practice in the other year before they graduate. After Mwati graduated as a primary school teacher, he taught at one of the primary schools in the Malawi defence force for two years before joining the secondary school teacher education program. He was the best student in class. At the time the data was generated, there were only four mathematics education students at the college. They were all males. In the previous years, the college had stopped recruiting students for both Diploma in Education and Bachelor of Education programs because of some logistical issues.

I generated data using video lesson recordings. Using video allowed me to record the events thoroughly. It also allowed me to look at the episode multiple times for further analysis (Girden & Kabacoff, 2011). The lesson was for grade 11 (upper secondary school). It was about solving exponential equations. In the final semester of the final year of study, students at Mwati's institution are given an opportunity to practise teaching at a secondary school that is attached to the college. This teaching was, hence, part of Mwati's teacher training programme. Before beginning the data generation, I asked for permission from the principal of the concerned college, the head teacher and the mathematics teacher of the concerned class.

I analysed the data using thematic analysis (Ritchie & Lewis, 2003). Firstly, I transcribed the video recordings in whole. Then, I analysed a transcript of a selected episode. I chose the particular episode because some elements that Mwati displayed in this episode were unusual for the Malawi context. I conducted both inductive and deductive thematic analysis (Powell et. al., 2003; Creswell, 2012). Themes were developed deductively from the theoretical framework. During the initial coding, some themes that were not in the theoretical framework were emerging from the data. Some of these themes were incorporated into the theoretical framework while others were regarded as separate categories of the characteristics of preservice teacher‘s mathematics for teaching exponential equations (Boyatzis, 1998). To achieve credibility of the results, another researcher analysed the data. We agreed in at least 90% of all the themes, with no discussion between the researchers. Furthermore, the findings were read and critiqued by other researchers.

RESULTS AND DISCUSSION

Lesson summary

The lesson consists of one event of which the object of learning is solving exponential equations by converting them to quadratic equations. The event is divided into four sub-events with a new sub-event marked by a key purpose. The first sub-event focused on a review of solving exponential equations by equating exponents. Mwati started by asking students to evaluate $3^b \times 3^b$, $5^{2a} \times 5^1$ and 2^{x+1}. Students were required to use laws of indices to work out these problems. In the second sub-event, Mwati solved the equation $2^{2b} - 9 \times 2^b + 8 = 0$ which was discussed throughout the lesson. The equation was solved as a model example by the teacher. Mwati linked exponential equations to quadratic equations at a high level of abstraction. He also related the exponential equation to the quadratic equation $x^2 - 9x + 8 = 0$ and continued with solving the two parallel equations. Mwati's presentation of the solution process to the equation differed from his plan which indicated that he would let $x = 2^b$ so as to convert the equation to $x^2 - 9x + 8 = 0$. During his presentation of the example, the students seemed to follow how the teacher solved the two equations but they got lost in the middle and they explained to Mwati that they were confused. Despite the complaints raised by the students, Mwati gave them an exercise to solve in pairs. Sub-event 3 involved students practising the concept learnt by solving $5^{2x} - 6 \times 5^x + 5 = 0$. Few pairs solved the equation and one pair presented their work to the whole group. In sub-event 4, Mwati gave the students $3^{2n} - 6 \times 3^n + 9 = 0$ and $2^{2b+1} - 17(2^b) + 8 = 0$ to do as homework. I analysed the second sub-event and the results are discussed in the forthcoming paragraphs.

Mathematics for teaching

The mathematics for teaching that Mwati demonstrated includes explanations, representations, questioning, working with students' ideas and restructuring tasks. There were no definitions of terms in the lesson hence definitions are not part of the analysis in this paper. The selected examples are not evaluative of the preservice teacher but help elucidate the knowledge for teaching demands and challenges faced by the preservice teacher.

Explanations

In this study, any attempt to provide understanding of a problem to students (Brown & Armstrong, 1984) was regarded as an explanation. During the teaching and learning process, Mwati gave several explanations to enhance students' understanding of the methods of solving exponential equations. Some explanations were comprehensible and useful for students unlike others. Firstly, Mwati explained that in an exponential equation, when the bases are the same, the powers are equal. Secondly, he displayed ability to connect a topic to prior years. He reduced an exponential equation to a quadratic one. When explaining about the use of quadratic equations to solve $2^{2b} - 9 \times 2^b + 8 = 0$, he stated that 2^b should be regarded as the unknown. The following transcript of the second sub-event event provides explanations that Mwati gave about solving exponential equations:

T: Let us consider the equation $2^{2b} - 9 \times 2^b + 8 = 0$ and the term 2^{2b}. Let us break it like the way we did with $3^b \times 3^b = 3^{2b}$. So what will it be equal to?

Ralph: It will be $2^{2b} = 2^b \times 2^b$.

T: It will be $2^{2b} = 2^b \times 2^b$ or $2^{2b} = (2^b)^2$. Does that make sense?

Students: Yes.

T: Ok. Let us look at the laws of indices.

(He brought the chart with the laws of indices that he used on first and second day).

T: Law number 3. They are saying $(x^a)^b = x^{ab}$. Like in the problem we are solving $2^{2b} = (2^b)^2$. Therefore, the equation will be $(2^b)^2 - 9(2^b) + 8 = 0$ Now, from there, do you remember having done quadratic equations?

Students: Yes.

T: What we did on quadratic equations can be applied here. So we can make the equation be $x^2 - 9x + 8 = 0$. Have you seen that?

Students: Yes.

T: Therefore, this 2^b has to be taken as unknown. Therefore, we can solve it as a quadratic equation because this 2^b has been taken as unknown. Now, let us proceed working on $(2^b)^2 - 9(2^b) + 8 = 0$. What can we do from here? If you are given an equation like $x^2 - 9x + 8 = 0$. What is the next step here?

Rose: We need to factorise.

T: How can we factorise this one?

Charles: We can factorise by $x^2 - 8x - x + 8 = 0$.

T: Yes. Borrowing this information, can you apply it on $(2^b)^2 - 9(2^b) + 8 = 0$?

George: $(2^b)^2 - 8(2^b) - 2^b + 8 = 0$.

T: Do we all agree?

Students: Yes (some), No (others).

T: Let us agree. I was saying we need to apply the same knowledge we know about $x^2 - 8x - x + 8 = 0$ and apply that knowledge to $(2^b)^2 - 9(2^b) + 8 = 0$. Now this 2^b has to be taken as unknown. Now from here, we can factorise the equation because we have $2^{2b} = 2^b \times 2^b$. So we have $2^b(2^b - 8) - 1(2^b - 8) = 0$. Have you seen that?

Students: (Complaining, murmuring).

T: Look at $x^2 - 8x - x + 8 = 0$. Factor out x to have $x(x - 8) - 1(x - 8) = 0$.

[The teacher solved 2 parallel equations]

Students: We are confused.

T: This ($x(x - 8) - 1(x - 8) = 0$) is more or less like ($2^b(2^b - 8) - 1(2^b - 8) = 0$).

Yohave: I did not understand the second step. Where has the 9 gone?

T: At the second stage, where is the 9? [The teacher used $x^2 - 8x - x + 8 = 0$].

Future: $-8x - x$.

T: Here in $(2^b)^2 - 8(2^b) - 2^b + 8 = 0$, the 9 has been substituted by $-8(2^b) - 2^b$. That means if we add or subtract $-8(2^b) - 2^b$ we will get $-9(2^b)$. Have you seen that?

Students: Yes (some), (Others did not answer).

T: Ok. Let us proceed. Now from $x(x - 8) - 1(x - 8) = 0$, we need to factor out the bracket $(2^b - 8)$. This gives us $(2^b - 8)(2^b - 1) = 0$. This is the same as in $x^2 - 8x - x + 8 = 0$. In this one, we have $(x - 8)(x - 1) = 0$. So, we have either $x - 8 = 0$ or $x - 1 = 0$. Isn't it? Now, taking the same knowledge to our exponential equation, we have ether $2^b - 8 = 0$ or $2^b - 1 = 0$.

Mwati continued solving the equations $2^b - 8 = 0$ and $2^b - 1 = 0$ until he found that either $b = 3$ or $b = 0$. He seemed to have selected the methods of solving exponential equations with some insight and sophistication. However, he did not appear to be able to explain why the methods worked.

Representations

There are five elements of knowledge of representations which have been identified by Ball and her colleagues (Ball et. al., 2008). These include representing ideas skillfully, mapping physical, graphical, symbolic notation, and operations, making connections among representations, recognising what is involved in using a particular representation, selecting representations for particular purposes and making and using mathematical representations effectively. As may be observed from the transcript, the representations used by Mwati were verbal and written symbols. He attempted to use questions to connect the verbal and written representations but he was not that successful because he asked low order questions. Hence, it was difficult for students to understand the method of solution of exponential equations under discussion.

Questioning

Analysis of questions was mainly based on Bloom's taxonomy (Krathwohl, 2002). In this lesson, Mwati mostly asked factual and low order thinking questions. Other questions required 'yes' or 'no' answers. There was strong teacher guided instruction which made it difficult for students to construct their own comprehensive understanding of exponential equations. Mwati's questioning technique might reflect several issues. Firstly, it might be that emphasis was directed towards getting right answers. Secondly, it might reflect the teacher's procedural knowledge since he focused more on giving rules without reasons. It is also possible that he had not yet developed his questioning techniques. However, Mwati was a qualified primary school teacher and had taught for two years before he joined the secondary school teacher education programme. Moreover, he was a high academic achiever. As such, I would argue that he had limited ability to ask higher order questions and that he focused on procedural aspects of equation solving.

Working with students' ideas

Working with students' ideas involves interpreting and making mathematical and pedagogical judgements about learners' questions, solutions, problems and insights, responding productively to learners' questions and assessing students' mathematical learning and taking the next steps (Ball et. al., 2008). Of these, Mwati used questions to engage with students' ideas. As discussed in the preceding paragraph, the questions that Mwati asked were of low order. He also used explanations which were mostly procedural. His explanations dominated the lessons such that students participated mostly by responding to "yes" or "no" questions. The transcript above reveals that only one student asked a question which Mwati answered by explanation.

Another important aspect that Mwati seems to lack is anticipating students' thinking. An important aspect of planning a lesson is engaging in solving the lesson problem in various ways. This enables teachers to anticipate student thinking and the multiple ways they will devise to solve the problem. This also enables a teacher to plan the possible questions he/she may ask to stimulate thinking and deepen students' understanding (De Garcia, 2011). The findings, thus, reveal that Mwati displayed limited ability to work with students' ideas. For example, he ignored students' responses, questions, and solutions and was not flexible enough to change the direction of the lesson to take into account the points that arose.

Restructuring tasks

According to Ball et. al. (2008), restructuring tasks involves modifying tasks to make them easier or more difficult, making judgments about the mathematical quality of instructional materials and modifying as necessary and finding appropriate mathematical examples. When Mwati related $\left(2^b\right)^2 - 9\left(2^b\right) + 8 = 0$ to $x^2 - 9x + 8 = 0$, he attempted to restructure the exponential equation to a quadratic form that students were familiar with. He gave explanations while he solved the two equations at the same time but students could not understand. One of the statements by the teacher indicates that he realised that students did not understand but he seemed not to have a remedy to the problem hence he forced them to write an exercise. Thus, he was not flexible enough to make the solution process for $\left(2^b\right)^2 - 9\left(2^b\right) + 8 = 0$ to be transparent for the students so that they could generalise from it. It could thus be expected that he was not in possession of knowledge necessary for teaching exponential equations.

The findings in this study illuminate the results of a study conducted by Adler and Davis (2006) who found out that compression or abbreviation in contrast to unpacking of mathematical ideas was dominant among teachers. Similar results were also found from a study conducted by Huang (2012) in the USA and China. Huang found out that the participants had relatively limited knowledge of algebra for teaching.

CONCLUSION

In this study, Mwati’s mathematics for teaching was investigated. Findings reveal that Mwati selected the method of solving exponential equations with some insight and sophistication. However, he presented the method to students at a high level of abstraction. This made him unable to unpack or decompress equation solving to make it accessible to students. Since this difficulty could be observed in a participant who was particularly good in class, it would indicate that his mathematics teacher education programme does not focus on mathematics for teaching. This might imply that teacher education needs to be adjusted to focus more on mathematics for teaching. With sufficient mathematics for teaching, teachers will be able to unpack or decompress mathematical ideas so that they are accessible to students. Teachers will also be able to interpret students’ thinking, identify misconceptions, provide tasks and pose questions that will guide students’ interpretations of mathematics. The teachers will also be able to find appropriate strategies for inducing cognitive conflict that will help students to deconstruct their naïve theories and reconstruct correct mathematical conceptions.

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