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This study investigated a preservice secondary school teacher's pedagogical content knowledge (PCK) for teaching algebra. Data were generated using video-recorded interviews and analysed using thematic analysis. Findings indicate that content knowledge was influential in the preservice teacher's PCK. Secondly, the preservice teacher, who was one of the best students in his class, displayed some knowledge of analysing students' errors and anticipating their possible misconceptions. He appeared to be familiar with some methods of handling students' errors and misconceptions. However, it appears that he was not prepared to apply such methods in his teaching of algebra.

Keywords: Pedagogical content knowledge, preservice teacher, secondary school, algebra.

Introduction

A body of research indicates that teacher knowledge influences the quality of their teaching and student learning (Hoover, Mosvold, Ball, & Lai, 2016). Although there appears to be general consensus that mathematics teachers need to know the content in ways that surpass the knowledge of educated people outside the teaching profession (Ball, Thames & Phelps, 2008), more research is needed in order to investigate the types of knowledge needed for teaching particular mathematical topics at particular levels (Hoover et al., 2016). From her review of literature on teaching and learning of algebra, Kieran (2007) suggests that researchers have barely begun to investigate the knowledge needed for teaching algebra. Some studies have contributed to this area of research. For instance, Bair and Rich (2011) investigated the development of specialised content knowledge for teaching algebra among primary teachers. In the present study, we contribute to the field by investigating pedagogical content knowledge (PCK) for teaching algebra in secondary school.

Our study was carried out in Malawi – a country in southern Africa that experiences severe challenges in the education system. Secondary school students' performance in national examinations continues to be poor with only around 50 percent of students passing end-of-cycle examinations (Ministry of Education Science and technology, 2008), and students' performance in algebra is poor (Malawi National Examinations Board, 2008-2013). While explanations have been proposed concerning system failure, (Ministry of Education Science and Technology, 2008), we suggest that further investigations of teacher knowledge as a potentially relevant factor of influence is necessary. As such, the aim of this study was to investigate a Malawian preservice secondary school teacher's PCK for teaching algebra. Possible implications are discussed.

Theoretical framework

Two constructs guide the theoretical framework for this study: mathematical knowledge for teaching (Ball et al., 2008) and algebraic thinking (Kriegler, 2007). Ball et al. (2008) distinguish between three sub–categories of subject-matter knowledge and thus extend Shulman's (1986) original category. *Common content knowledge* refers to a kind of mathematical knowledge and skill that is used in settings other than teaching (Ball et al., 2008). *Specialised content knowledge*, on the other hand, refers to mathematical knowledge and skill that is unique to teaching. In addition, they present *horizon content knowledge* as a third category of subject-matter knowledge. For sake of simplicity, we focus broadly on content knowledge in this study. Ball et al. (2008) also distinguish between three sub-categories of PCK. *Knowledge of content and students* (KCS) is knowledge that combines knowing about students and knowing about mathematics. *Knowledge of content and teaching* (KCT) combines knowledge about teaching and knowledge about mathematics. Finally, Ball et al. (2008) present *knowledge of content and curriculum* as a third sub-category of PCK. In the present study, we focus on content knowledge and two sub-categories of PCK: KCS and KCT.

The second construct of the theoretical framework draws upon Kriegler's (2007) work on mathematical thinking tools of the algebraic thinking framework. Kriegler asserts that mathematical thinking tools are analytical habits of mind. They are organised around three topics: problem solving skills, representation skills, and quantitative reasoning skills. Teachers should be able to solve algebra problems using multiple approaches and problem solving strategies. They should be able to translate among representations and solve problems inductively and deductively.

Method

The study reported here is part of a larger qualitative study that investigates four preservice secondary school teachers' mathematical knowledge for teaching (see Mamba, 2016a; Mamba, 2016b). In previous publications, the first author reported on results from a task based interview that she conducted with one preservice secondary school mathematics teacher and video lesson observation for one lesson. In the current paper, we explore PCK for teaching algebra, considering the case of Dinga (pseudonym). Dinga was a diploma in education student in a three-year programme at a college of education in Malawi. When data for this study were generated, he was in the final year of study. Being a high-achieving student in mathematics in his class, he was considered an "information-rich" case for in-depth study (Yin, 2014). Video-recorded semi-structured interview was used to generate the data, allowing for multiple, in-depth rounds of analysis of the data (Girden & Kabacoff, 2011). Interview tasks reported here were adapted from a Malawi secondary school mathematics textbook (Gunsaru & Macrae, 2001) and were piloted before the main study.

The first author conducted the interview and transcribed the video recordings. These transcripts were analysed using a combination of inductive and deductive thematic analysis (Yin, 2014). Analysis of the interview transcripts involved identification of the PCK that Dinga displayed as he answered the interview questions. The themes that guided analysis were developed *a priori* from the theoretical framework. Themes for content knowledge were problem solving skills, representation skills, quantitative reasoning skills and justifying. Themes for KCS included predicting students' errors and misconceptions, understanding reasons for errors and misconceptions, and asking questions to reveal or understand students' reasoning and misconceptions. KCT was coded into methods of handling students' misconceptions and knowledge of instructional tasks to be used to enhance conceptual understanding and choosing instructional strategies. Some themes that were not in the theoretical framework were developed *a posteriori* from the data. The themes were further grouped into three categories: CK, KCS and KCT. To achieve credibility of the results, another researcher analysed the data. In all cases we got at least

90% agreement, with no discussion between the researchers. Furthermore, the findings were read and critiqued by other researchers.

Results and discussion

In the following, we present illustrative examples of results from the analysis of data from the video-recorded interview.

Content knowledge

During the interview, Dinga was asked to solve the following equation: $x^2 = 2x + 8$. He solved the equation using two approaches: factorisation method and the quadratic formula. When asked about other methods apart from the two he used, Dinga explained that he had forgotten the other method. His explanation showed that he had knowledge of completing the square, but he had forgotten how to use this method for solving quadratic equations. Dinga's solution processes revealed that he had procedural knowledge, since he did not display knowledge of the conceptual foundations of quadratic equations. For instance, when he solved the equation $x^2 = 2x + 8$ by factor method and quadratic formula, his difficulties of explaining the procedures indicated that he knew the "what" of the procedure but not the "why". For mathematics teachers, knowing both the "what" and the "why" of a procedure is important (Shulman, 1986). Dinga's knowledge of content therefore seemed limited in depth and breadth. Inability to solve the equation, using the other approaches, like completing the square and graphing, also indicated limitations in problem solving skills and representation skills. When solving the equation using the two methods he remembered, however, Dinga used rules of logic to come up with next steps in the procedures he used – thus indicating skills in deductive and quantitative reasoning.

When answering questions in Task 2 (Figure 1), he did not attempt to interpret the graph first, although the interviewer asked him to do so. This also indicates limitations in content knowledge and algebraic thinking skills.

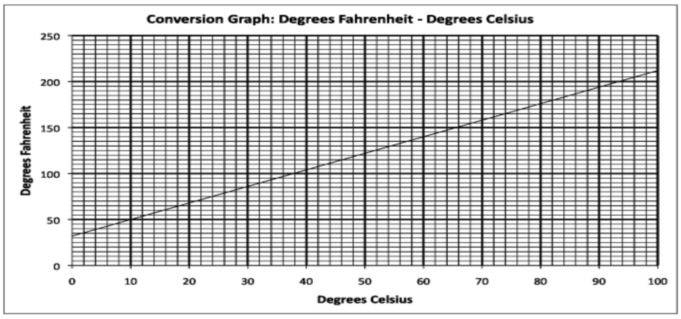
Knowledge of content and students

Dinga displayed knowledge of predicting methods that students may find easy or difficult, predicting students' errors and misconceptions, understanding reasons for misconceptions, and asking questions to reveal or understand students' reasoning and misconceptions. For instance, the interviewer asked Dinga to assume that he gave this equation $x^2 = 2x + 8$ to his algebra students to solve. When asked to explain what methods he thought his students would use, Dinga explained that his students would use factorisation because, to him, factorisation method is easy and the other methods are difficult. Our interpretation of Dinga's response is that he based his prediction of what students would find difficult on his own experienced difficulties.

The second task in the interview involved interpretation of a conversion graph between °C and °F (see figure 1 below).

Task 2: Suppose you give your students the following problem:

Below is a graph of an equation $Tf = \frac{9}{5}(T_C) + 32$ where $Tc =$ temperature in degrees Celsius and $Tf =$ Temperature in degrees Fahrenheit. Use the graph to convert the following temperatures: (a) 10°C, 40°C and 77°C respectively; (b) 50°F, 100°F and 170°F respectively.



i. What error(s) may students exhibit as they answer this question?
ii. What underlying mathematical misconception(s) or misunderstanding(s) might lead the student to the error presented in this item? How might the student have developed the misconception(s)?
iii. What further question(s) might you ask to understand the student's misconception(s)?
iv. What instructional strategies and/or tasks would you use during the next mathematics lesson to address the student's misconception(s)? Why?

Figure 1: Conversion task

During the discussion extracted in the below excerpt from the transcripts, Dinga was first asked to read, solve and understand task 2. Then, the interviewer asked him to explain the errors and misconceptions that students might display as they attempt to answer this question.

Dinga: (...) They may misread the scale, or they may not understand the scale and they may come up with different values.

FM: Ummmm! Hummm!

Dinga: Some students don't know what horizontal axis is and what vertical axis is. So in this kind of problem, they can easily do the reverse.

FM: What misconceptions might lead to the errors you presented in this item?

Dinga: Sometimes, they put forward the belief that mathematics is difficult. So they may think that they may not manage.

FM: What else apart from the belief that mathematics is difficult?

Dinga: Teachers also contribute. If a teacher does not understand a topic, he/she does not teach it. He/she teaches a topic that is easy.

By explaining that students will misread scale and come up with different values from the expected values due to misunderstanding of scale, and that students may misallocate a point on the coordinate system, Dinga displayed ability to predict errors students may exhibit as they interpret that graph. However, when Dinga said, "if students do not understand scale, they would find some difficulties about how to come up with a conversion graph", he changed the purpose of the task from interpreting the graph to drawing the graph. Secondly, although Dinga suggested that graph interpretation is easier for Malawian students than drawing the graph, he did not interpret the graph himself, hence what he said contradicted with what he did during the interview. By explaining that graph interpretation is easier than drawing graphs, Dinga displayed limited knowledge of levels of graph interpretation. According to Cursio (1987), graph interpretation is a cognitive task involving three levels of understanding, namely reading the data, reading between the data and reading beyond the data. Dinga's understanding of these levels may influence students' development of graph interpretation abilities. Thus, Dinga displayed limited ability to predict students' difficulties about interpreting a linear graph – possibly because he struggled to interpret the graph himself. Failure to interpret the graph was also an indication of limitation in representation skills. He was unable to interpret information within a representation. We noted that while algebra students often create graphs from equations, they rarely practice creating equations from graphs and interpreting the graphs – hence Dinga's difficulty.

When asked to explain the misconceptions that may lead to the errors mentioned, Dinga pointed out beliefs and teachers. By explaining that teachers also contribute to errors and misconceptions, Dinga proposes that some misconceptions originate from experiences in school – students' interaction with teachers being one of the experiences. However, his responses indicate that he did not know which causes the other. He identified the errors but was not in a position to understand the misconceptions that could be possible causes of such errors. Instead, Dinga explained the causes of misconceptions. This confusion might have resulted from lack of understanding of errors, misconceptions and their causes. Understanding of each is important for the teacher, because, as with the weeds, the roots must be tackled if the weeds are to disappear. Similarly, teachers must deal with the causes of errors and misconceptions to help students overcome them.

To understand or reveal students' reasoning and misconceptions, Dinga explained that he would ask the students about the meaning and interpretation of scale, and he would identify the misconceptions from their responses. He also explained that he would ask the students to tell him what the horizontal and the vertical axes represent, because some students do not know what horizontal and vertical axes represent. By explaining that students confuse between horizontal and vertical axes, Dinga displayed understanding of students' confusion between independent and dependent variables. Asking him to give examples of probing questions that he said he would ask, we expected that he would mention questions like "explain your answer", "why do you think so?", "How did you get that?" Dinga did not suggest such questions. We interpret this as an indication of limited knowledge of what questions to ask in order to identify misconceptions.

Knowledge of content and teaching

The KCT that Dinga displayed included predicting strategies for teaching how to solve the quadratic equation $x^2 = 2x + 8$, and strategies for handling students' errors and misconceptions. When asked how he would teach solving quadratic equations using the equation $x^2 = 2x + 8$, Dinga explained that he would give them steps to follow for them to come up with the roots of the equation. The procedural knowledge he displayed when solving the equation might have influenced this response. Dinga also explained that he would ask volunteers to solve the equation on the chalkboard in any way they want in order to understand the way students understood the problem. In this case, Dinga decided to use a hybrid of teacher centred and student centred teaching methods. Although, this might be an improved version of teacher centred strategies, Dinga revealed knowledge of teaching the "what" but not "why". Thus, Dinga displayed lack of an important aspect of subject matter knowledge: "knowing why". Even and Tirosh (1995) argue that "knowing that" is not enough, and they suggest that "knowing why" enables the teacher to make better pedagogical decisions. It could thus be expected that Dinga lacks the knowledge necessary for teaching equations effectively to his students. In the closing statements of the interview, Dinga explained his limitations with lack of teaching experience.

In order to handle students' errors and misconceptions, teachers need to use appropriate strategies to create moments of cognitive conflict and help students resolve the conflict (Sayce, 2009). Dinga pointed out that he would use discussion and group work. The transcript below illustrates this.

FM: What instructional strategies would you use to address the misconceptions you have mentioned?

Dinga: I will use discussion, grouping, … (silence).

FM: How could these methods work in addressing these misconceptions?

Dinga: If I put my students in groups of 5 or 10, those who understood may assist others.

FM: Um! Humm!

Dinga: I will also give a summary of the topic so that students can easily correct their mistakes.

FM: How else can you address the misconceptions?

Dinga: Maybe giving my students a lot of exercises to do on the topic which they have problems so that the students understand the concept.

The results in the transcript above concur with Sayce (2009) who asserts that, to induce cognitive conflict, teachers should encourage collaborative working, especially in mixed ability groups. However, Dinga did not reflect on any features of group work that may facilitate this. He also did not reflect on the possibility that cognitive conflicts might be introduced by peers, but he seemed to suggest that the less able students could be passive recipients of the other students' explanations. We also argue that since errors and misconceptions are deeply rooted erroneous conceptions, summarising content covered in a lesson or giving students more exercises might neither be the root to cognitive conflict nor the way out of the conflict. The possible cause for Dinga's limitations in methods of handling students' errors and misconceptions might be that he did not solve the mathematical task (task 2) – possibly because he seemed not to know what it takes to interpret graphs (Cursio, 1987). A teacher needs to be able to solve the mathematical task before presenting it for the students in the classroom. Solving the task enables the teacher to anticipate students' solution methods, errors, misconceptions and questions to ask the students.

Conclusion

In this study, we investigated the PCK for teaching algebra displayed by one preservice secondary school teacher – Dinga. The findings reveal that content knowledge was the overriding determinant of Dinga's KCS and KCT. For instance, Dinga explained that he would teach solving the equation $x^2 = 2x + 8$ by giving the students a procedure to follow to solve the equation, probably because he lacked the "why" of the procedure and he solved the equation likewise. He also had some difficulties predicting students' errors and misconceptions because he was unable to interpret the graph. These findings support the fact that PCK involves knowledge and skills that are highly interrelated to each other (Even & Tirosh, 1995). Thus, teachers should possess in-depth content knowledge, have a rich repertoire of teaching strategies to promote students' understanding of a particular topic and to understand and handle students' errors and misconceptions (Kilic, 2011). The findings also support the fact that preservice teachers possess limited PCK (Kilic, 2011). Although Dinga appeared to be familiar with some methods that can be used to handle students' errors and misconceptions, he seemed to lack conceptual approaches to students' errors and misconceptions because of his limitations in content knowledge and the "why" of his ideas.

It is also worth noting that Dinga's responses to the interview questions might be influenced by several factors. Firstly, it might be that Dinga might be able to solve equations but might not be able to explain his ideas. The fact that the interview tasks were adapted from a textbook might also limit Dinga's explanations and responses during the interview if the textbook was not familiar to him. But I gave the preservice teachers the textbooks a week before the commencement of the data collection for them to study in preparation for the tests and interviews. Furthermore, Dinga's responses might be influenced by the secondary school mathematics curriculum he went through as a mathematics student. The mathematics curriculum did not give students opportunities to explain their reasoning during mathematical problem solving. The way mathematical problem solving is handled in preservice teacher education in Malawi might also influence the results from the interview with Dinga. For instance, there is less practical work on mathematical problem solving in teacher education, yet the preservice teachers are expected to teach their students how to use the approach during teaching practice.

The results from this study cannot be generalized to all preservice teachers at Dinga's institution. Further research with a larger sample needs to be carried out to find out whether results from this case study are generalisable at a large scale. In addition, the measurement of Dinga's PCK was somewhat constrained due to the limitation of not having access to classroom students. In his responses, Dinga had to 'work' within a hypothetical situation. While his limitations in PCK were more evident during the task-based interview, it is possible that additional evidence of PCK could be obtained through a variety of problems and video lesson observations. These results, however, help us to learn that developing PCK and problem-solving skills among preservice secondary teachers by "providing teachers with opportunities to learn mathematics that is intertwined with teaching" (Hoover et al., 2016, p. 12) is crucial for mathematics teaching and learning.

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